ESM 121

Water Science
and Management

Exercise 7:

Risk Analysis

and

Expected Monetary Value

Samuel Sandoval Solis, PhD

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# Objective

The objective of this exercise is to provide a set of exercises of a benefit-cost analysis that consider risk analysis by using the expected monetary value method.

## *Useful Formulas*

Expected Monetary Value

$$E[X] = \sum_i x_i p_X(x_i)$$

Marginal Distribution

$$p_X(x) = \sum_y [p_Y(y) * p_{X|Y}(x \mid y)]$$

Conditional Distribution

$$p_{X|Y}(x \mid y) = \frac{p_{XY}(x, y)}{p_Y(y)}$$

Joint Distribution

$$p_{XY}(x, y) = p_Y(y) * p_{X|Y}(x \mid y)$$

# Expected Monetary Value

## *Exercise 1 (Adopted from Loucks and van Beek Problem 7.5).*

Table 1 shows the data of expected benefits and cost for recreation activities on a determined reservoir. Using the data provided in Table 1, on possible recreation benefits, cost (recreation losses).

1) Estimate the Net Benefits

Table 1.- Data of associated benefits and cost for the

| **Summer storage level (ft)** | **Probability of storage level** | **Average Benefits (thousand $)** | **Decrease in recreation benefits (Cost) (thousand $)** | **Net Benefits (thousand $)** |
|---|---|---|---|---|
| 200 | 0.1 | 10 | 5 | |
| 250 | 0.2 | 10 | 2 | |
| 300 | 0.4 | 10 | 0 | |
| 350 | 0.2 | 10 | 1 | |
| 400 | 0.1 | 10 | 4 | |

2) Estimate the *Expected* Net Benefits

## *Exercise 2 (Adopted from Loucks and van Beek Problem 7.5).*

Table 2 shows the data on irrigated agricultural yields for rice. The crop yield varies with respect to the water applied per hectare. The capital cost to grow a hectare of rice is $3,160. The current market price for a ton of rice is $325 per ton.

1) Estimate the benefits for each type of irrigation water allocation

2) Estimate the net benefits per acre for each water allocation

Table 2

| Water Allocation (acre-feet) | Probability of Allocation | Cost to grow rice ($/acre) | Crop Yield (ton/acre) | Market Price of Rice ($/ton) | Benefits ($/acre) | Net benefits ($/acre) |
|---|---|---|---|---|---|---|
| 1 | 0.2 | $3,160 | 6.5 | $325 | | |
| 2 | 0.3 | $3,160 | 10 | $325 | | |
| 3 | 0.3 | $3,160 | 12 | $325 | | |
| 4 | 0.2 | $3,160 | 11 | $325 | | |

3) Estimate the expected net benefits using the probability of allocation and the Net Benefits.

## Multiple Random Variables

### *Exercise 3 (Adopted from Robert Gilbert).*

Table 3 shows the joint distribution of the hours that a driller machine was tested and the productivity associated with that time.

1) Estimate the joint Probability Mass Function (also known as Joint Distribution or Relative Frequency) of the data.

Table 3

| Duration X (hr) | Productivity Y (%) | Productivity Y ($) | Number of Observations | Joint PMF $P_{X,Y}(x,y)$ |
|---|---|---|---|---|
| 6 | 50 | 750 | 2 | |
| 6 | 70 | 1050 | 5 | |
| 6 | 90 | 1350 | 10 | |
| 8 | 50 | 750 | 5 | |
| 8 | 70 | 1050 | 30 | |
| 8 | 90 | 1350 | 25 | |
| 10 | 50 | 750 | 8 | |
| 10 | 70 | 1050 | 25 | |
| 10 | 90 | 1350 | 11 | |
| 12 | 50 | 750 | 10 | |
| 12 | 70 | 1050 | 6 | |
| 12 | 90 | 1350 | 2 | |
| | | | 139 | |

2) Estimate the marginal distribution for the Duration (X-variable)

3) Estimate the conditional distributions for each productivity *given* a determined duration

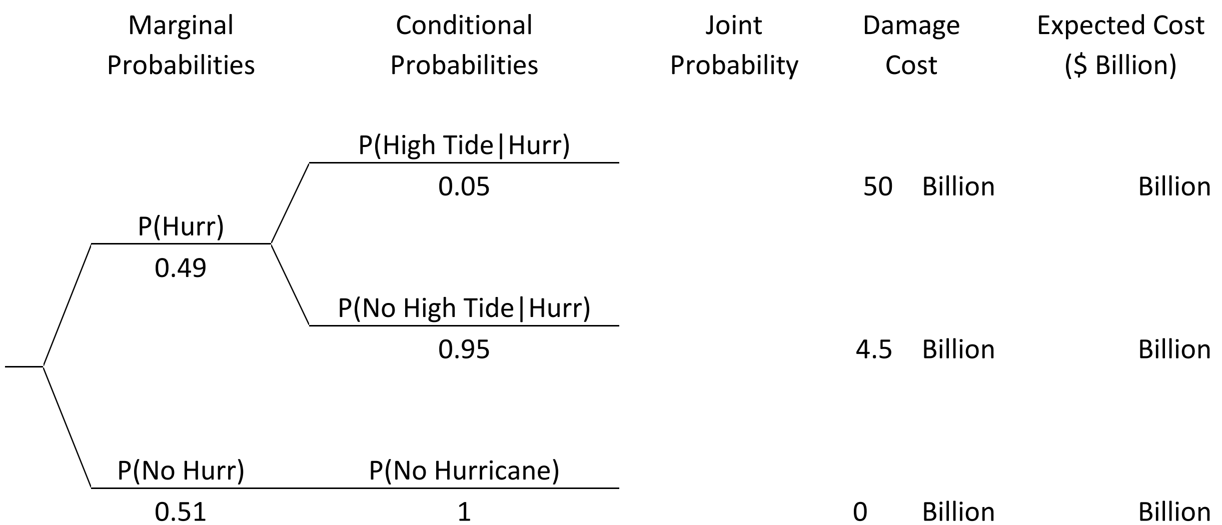
4) Build a decision tree with the marginal, conditional and joint distributions.

5) Estimate the expected value using the decision tree.

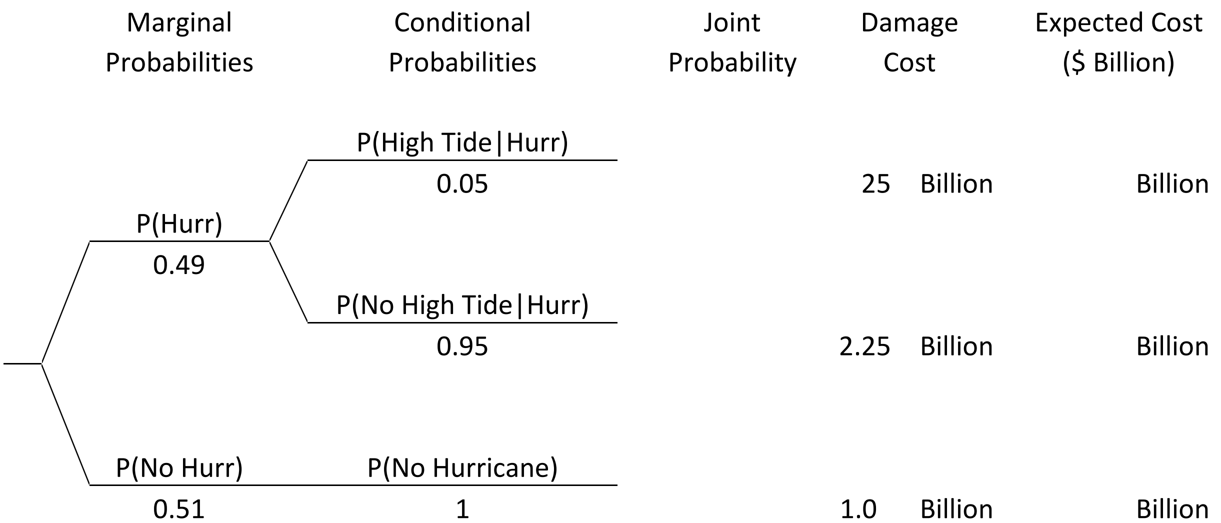
## *Exercise 4 (Adopted from Robert Gilbert).*

Last year, hurricane Sandy did a lot of damage to the East coast. According to NOAA, the probability that the East coast is hit by a hurricane is 0.49 each year. Also, the probability that a high tide occur during (given) a hurricane is very small, 0.05. According to experts, the estimated damage that Sandy caused is $50 Billion! The average damage that (typical) hurricanes cause is $4.5 Billion.

1) The following table decision tree show the Marginal and Conditional probabilities of the No-action Scenario, meaning, do not built structures to mitigate hurricane damages. Estimate the joint probabilities and the expected coast for this no action scenario.



2) Now, let’s consider the scenario where $1 billion is spent every year despite the fact that a hurricane happens or not (in the no “No Hurricane” branch ). Also, let’s consider that because of this investment the damage cost when Hurricanes happen (with and without Hide Tide) occur is reduced by half ($25 Billion in the High Tide|Hurricane and $2.25 Billion in the No High Tide|Hurricane). Estimate the joint probabilities and the expected coast for this no action scenario.




3) Compare the expected cost of both scenarios, which one is smaller? Which project would you select?