Water Resources Modeling
Optimization Modeling

Dr. Samuel Sandoval Solis
Professor and Specialist in Water Resources Management
UC Davis – UC Agriculture and Natural Resources

UCDAVIS WATER MANAGEMENT LAB

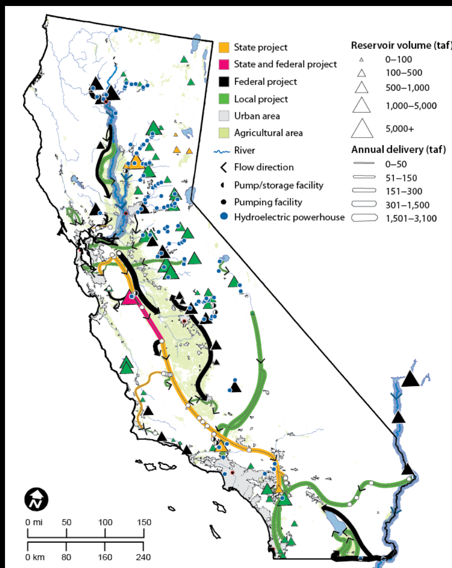
University of California
Agriculture and Natural Resources

UCDAVIS
UNIVERSITY OF CALIFORNIA

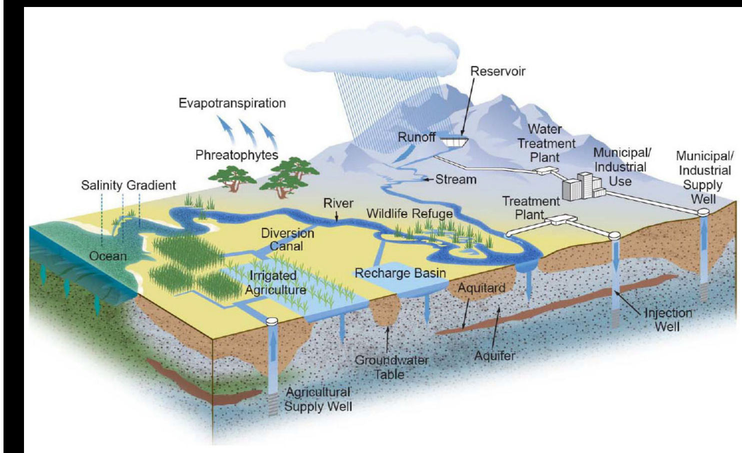
Outline

- Water Resources Modeling
- Optimization Modeling

Water Resources Systems

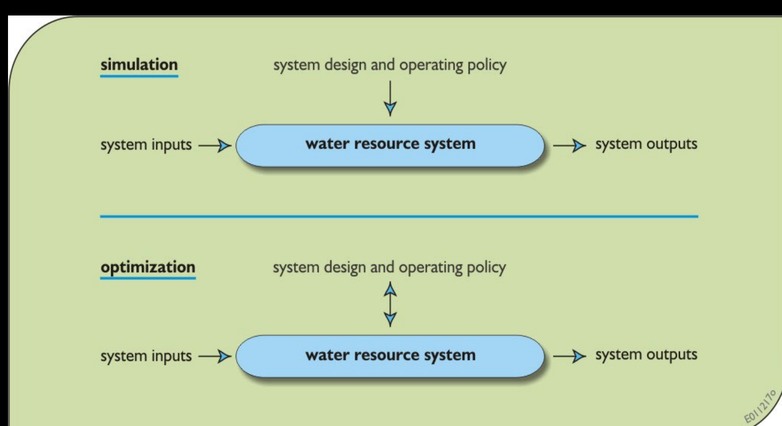


There is a need for tools



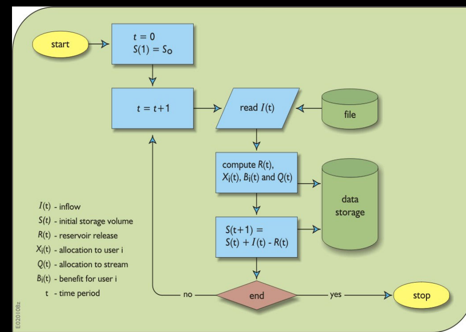
Simulation and Optimization

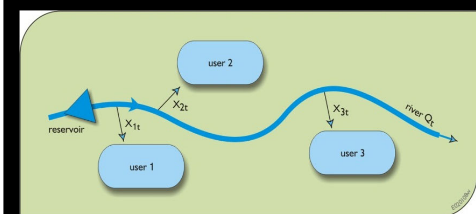
- Simulation models: Predict response to **given design**
- Optimization models: Identify **optimal design** or operation

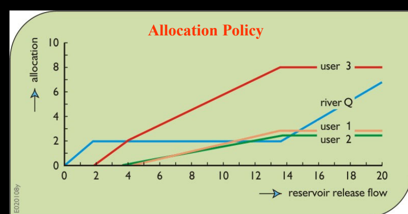


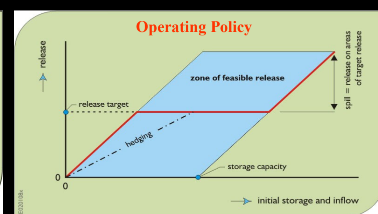
Simulation

- Address "**What if …**" questions
- What will likely happen
- Include larger hyd, econ, and env. data
- i.e. "evaluate change <u>given a design or policy</u>"



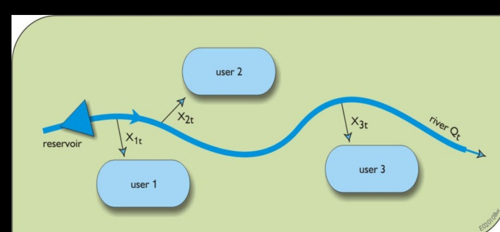






Optimization

- Used for design/insights:
  - "Maximize the Net Benefits …" or
  - "Minimize the shortages"
- Look for the optimized (best - ideal) operation
- Perfect foresight



Optimization model

Benefits: $B_i(x_{it})$

Decision Variables: x_{it}

Objective:

$$Maximize \sum_{t=1}^{T} \sum_{i=1}^{3} B_i(x_{it})$$

Constraints:

$$x_{1t} + x_{2t} + x_{3t} \le R_t \quad t = 1,2,\cdots$$

$$S_{t+1} = S_t + I_t - R_t \quad t = 1,2,\cdots$$

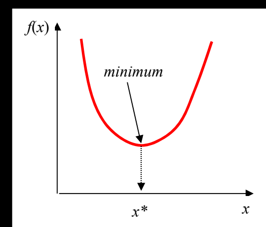
$$S_t \le K \quad t = 1,2,\cdots$$

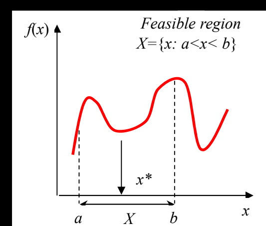
Optimization model

General concept

Identify the key variables (decision variables) that optimize (maximize or minimize) an desirable objective (function)

$$\text{minimize } f(\mathbf{x}) \quad \leftarrow \text{Objective function}$$
$$\mathbf{x} \quad \leftarrow \text{Decision variables}$$
$$\text{subject to}$$
$$\mathbf{x} \in X$$
$$Ax + b \leq c \quad \leftarrow \text{Constraint set}$$
$$x \geq 0$$





Steps for Linear Programming

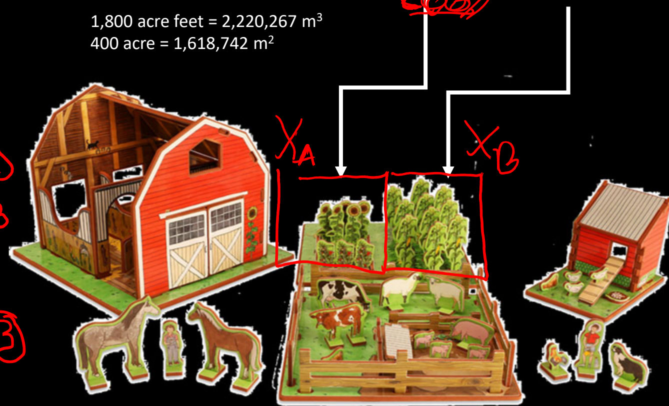
1. Identify the Objective Function and decision variables: Maximize or Minimize?
2. Define Objective Function (Write the Obj. Funct. Eq.)
3. Define the Constraints (Write the Constraints Eqs.)
4. Define the Feasible Region
5. Obtain the vertices of the feasible region
6. Substitute vertices in the Objective Function
7. Select the data where the value is Maximized/Minimized

Example 1

- Objective:
  Maximize the Profits
- Production Agriculture project
- 1,800 acre feet of water
- Decision Variables
  X_A = Acres of Crop A
  X_B = Acres of Crop B

	Crop A	Crop B
Water requirement (Acre feet/acre)	3	2
Profit ($/acre)	300	500
Max area (acres)	400	600

1,800 acre feet = 2,220,267 m^3
400 acre = 1,618,742 m^2



Max (Z)

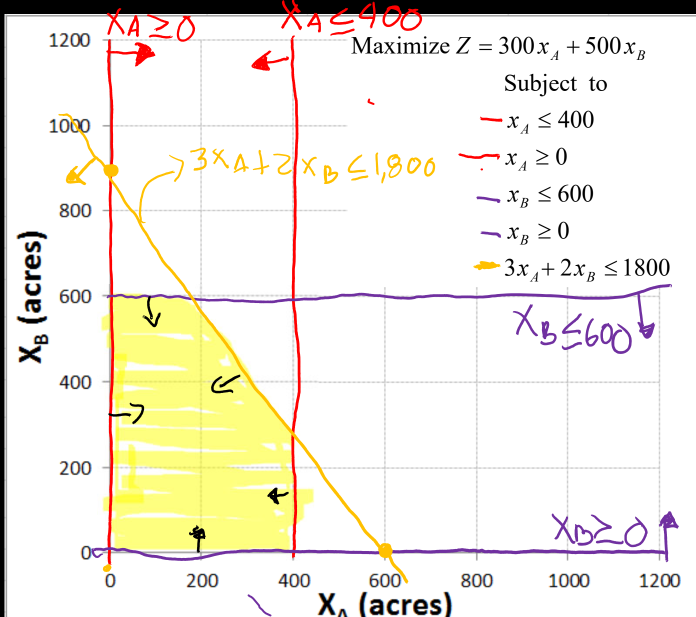
$Z = \text{Profits} = 300 * X_A + 500 * X_B$

$0 \le X_A$ $\quad 0 \le X_B$

$400 \ge X_A$ $\quad 600 \ge X_B$

$3X_A + 2 * X_B \le 1{,}800$

4) Define feasible region



$3X_A + 2X_B = 1{,}800$

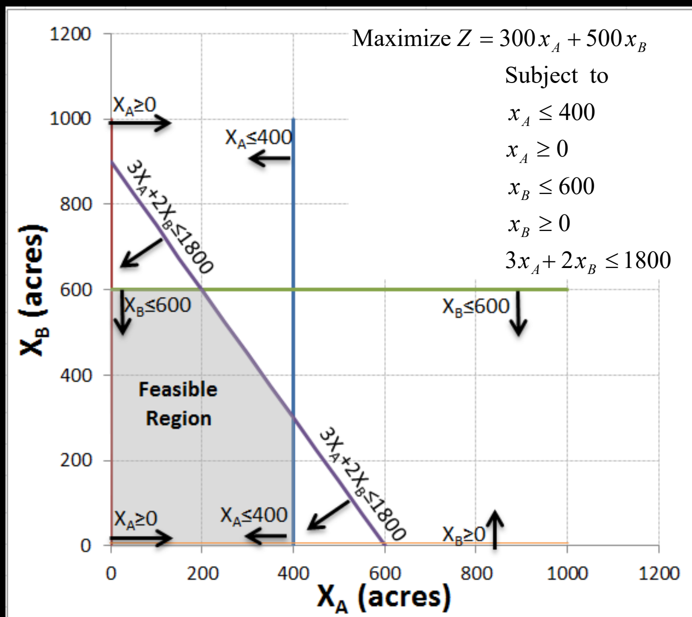
if $X_A = 0$

$2X_B = 1{,}800 \quad X_B = \frac{1{,}800}{2} = 900$

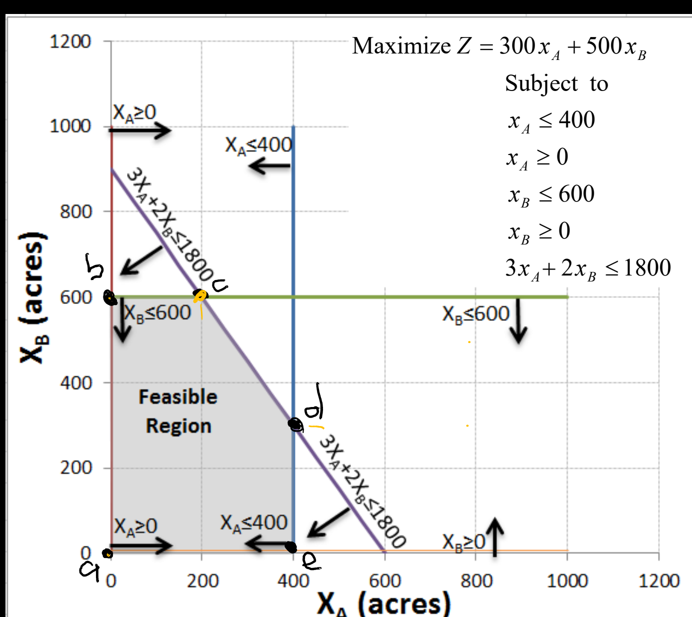
if $X_B = 0$

$3X_A = 1{,}800 \quad X_A = \frac{1800}{3} = 600$

4) Define feasible region



5) Obtain the vertices of the feasible region



Point	X_A	X_B
a	0	0
b	0	600
c	200	600
d	400	
e	400	0

$3X_A + 2X_B = 1,800$; when $X_B = 600$

$3X_A + 2(600) = 1,800$

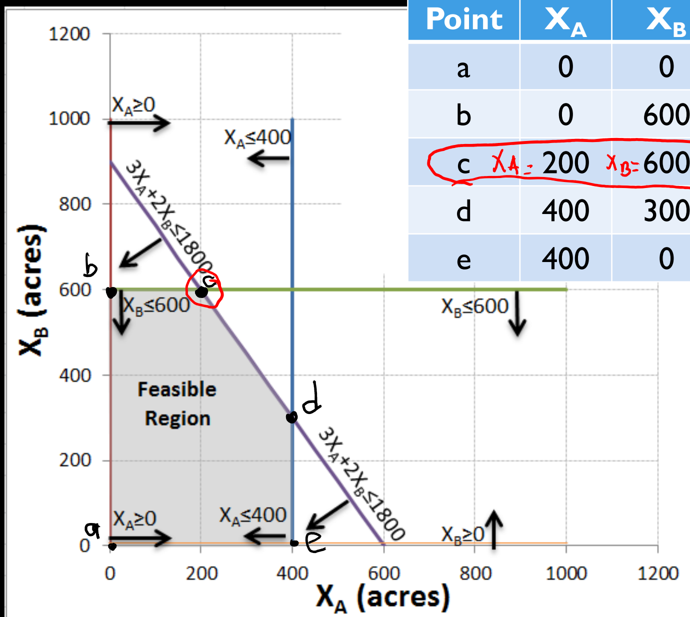
$X_A = \frac{1,800 - 1,200}{3} = \frac{600}{3} = 200$

when $X_A = 400$

$3(400) + 2X_B = 1800$

$X_B = 1800 - 1200$

6) Substitute values in the Objective Function
7) Find optimal design



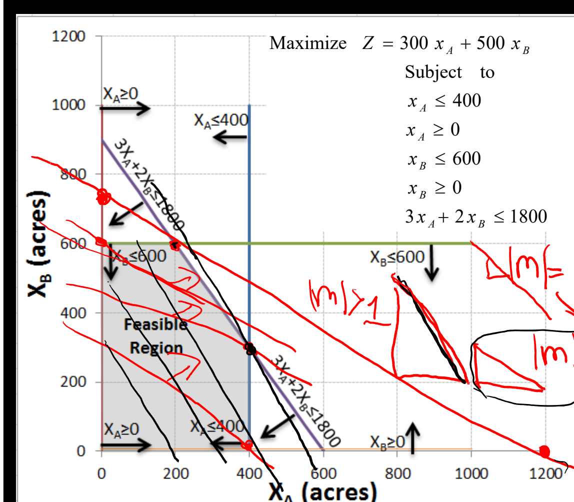
Point	X_A	X_B	$Z=300X_A+500X_B$ ← Benefits ($)
a	0	0	0 + 0 = 0
b	0	600	0 + 300,000 = $300,000
c	X_A = 200	X_B = 600	60,000 + 300,000 = $360,000 ✓
d	400	300	120,000 + 150,000 = $270,000
e	400	0	120,000 + 0 = $120,000

$$\text{Maximize } Z = 300x_A + 500x_B$$

Subject to

$$x_A \leq 400$$
$$x_A \geq 0$$
$$x_B \leq 600$$
$$x_B \geq 0$$
$$3x_A + 2x_B \leq 1800$$

X_A	X_B	$Z=300X_A+500X_B$ = 360,000
200	600	



Key Concepts

- Objective Function Slope

$300X_A + 500X_B = 360,000$

if $X_B = 0$ $X_A = 360,000/300 = 1,200$ — Intercept

if $X_A = 0$ $X_B = 360,000/500 = 720$ — slope

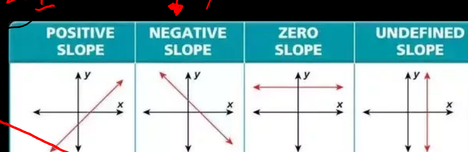
$300X_A + 500X_B = 360,000 \rightarrow Y = a + mx$ (intercept)

$X_B = \dfrac{360,000 - 300X_A}{500}$

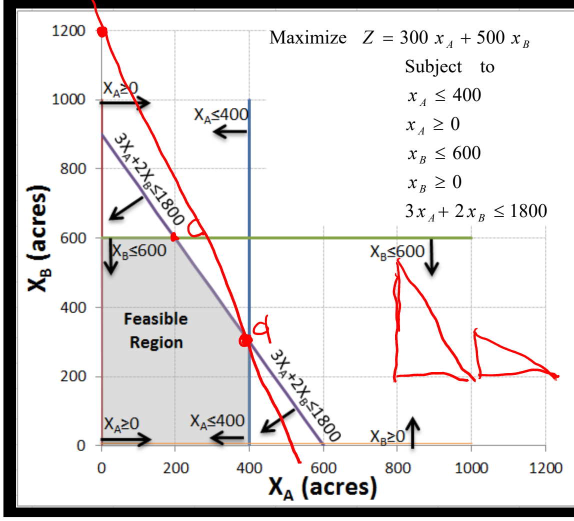
$X_B = 720 - 0.6X_A$ -0.6 $m = \dfrac{0.6}{1}$

$Y = a - mX$

(−)



X_A	X_B	$Z=500X_A+300X_B$
c: 200	600	100,000 + 180,000 = 280 k
d: 400	300	200,000 + 90,000 = 290 k



Key Concepts

- Objective Function Slope

$500X_A + 300X_B = 290{,}000$

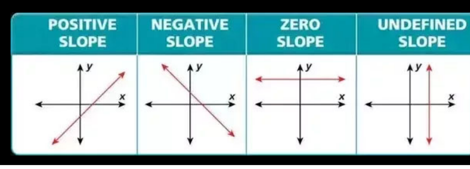
$Y = a + mX$

$X_B = \frac{290{,}000 - 500X_A}{300}$

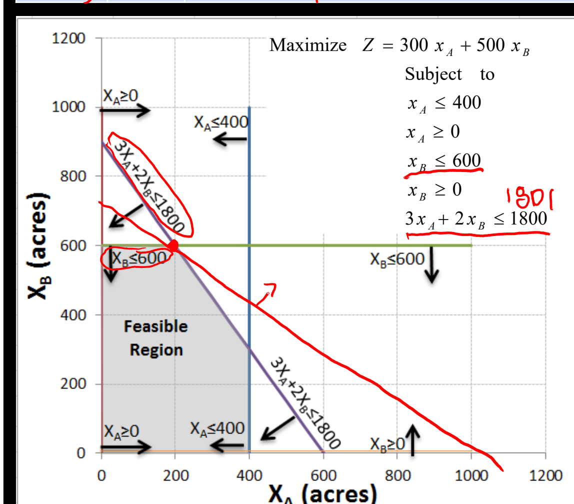
$X_B = 1{,}200 - 1.67X_A$

$Y = a + mX$

$|m| > 1$



X_A	X_B	$Z=300X_A+500X_B$
200	600	= 360,000
199.33	601	= 360,300
200.33	600	= 360,100



Key Concepts

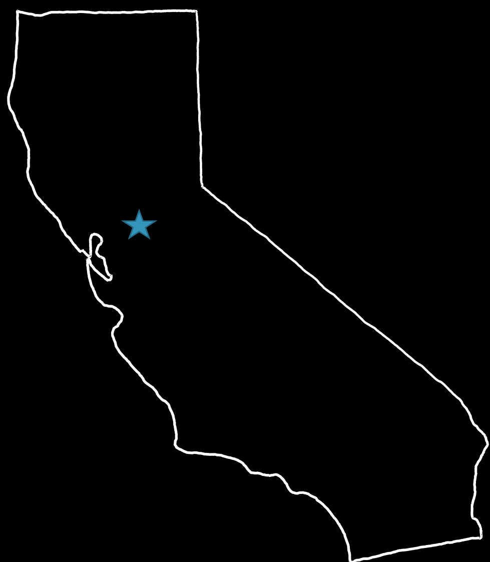
- Constraints:

Binding	Non Binding
2 $(x_B \le 600)$ = \$300	$x_A \le 400$
	$x_A \ge 0$
2 $(3x_A + 2x_B \le 1800)$ = \$100	$x_B \ge 0$

- Lagrangian Multipliers (Shadow Price)

SP = \$300 $X_B \le 600$ Increase Crop B

SP = \$100 $3X_A + 2X_B$ increase Water



Thank you

samsandoval@ucdavis.edu

watermanagement.ucdavis.edu

eflows.ucdavis.edu

University of California
Agriculture and Natural Resources

UC DAVIS WATER MANAGEMENT LAB