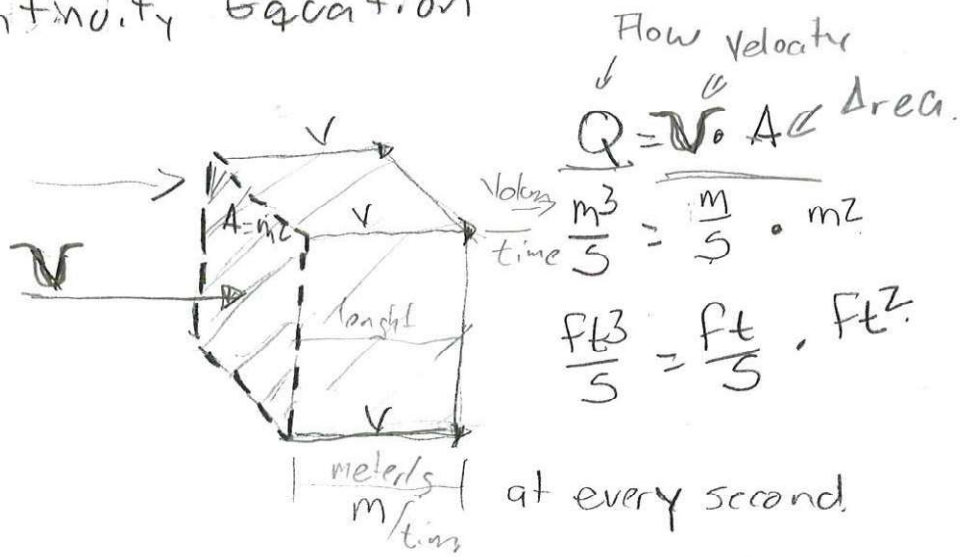
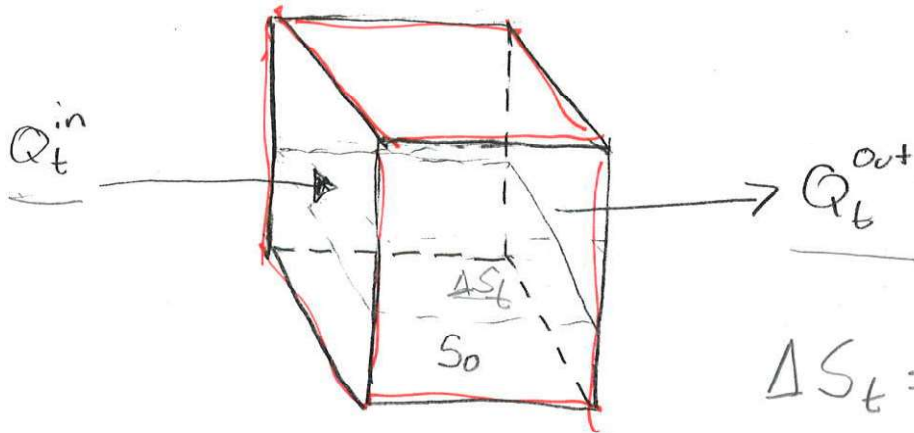


1) Continuity Equation



Control Volume



$$\Delta S_t = I_t - O_t$$

$$\Delta S_t = Q_t^{in} - Q_t^{out}$$

$$\Delta S_t = I_t - O_t$$

$$S_t - S_{t-1} = I_t - O_t$$

$$S_t = S_{t-1} + I_t - O_t$$

For $t=1$

$$S_1 = S_0 + I_1 - O_1$$

For $t=2$

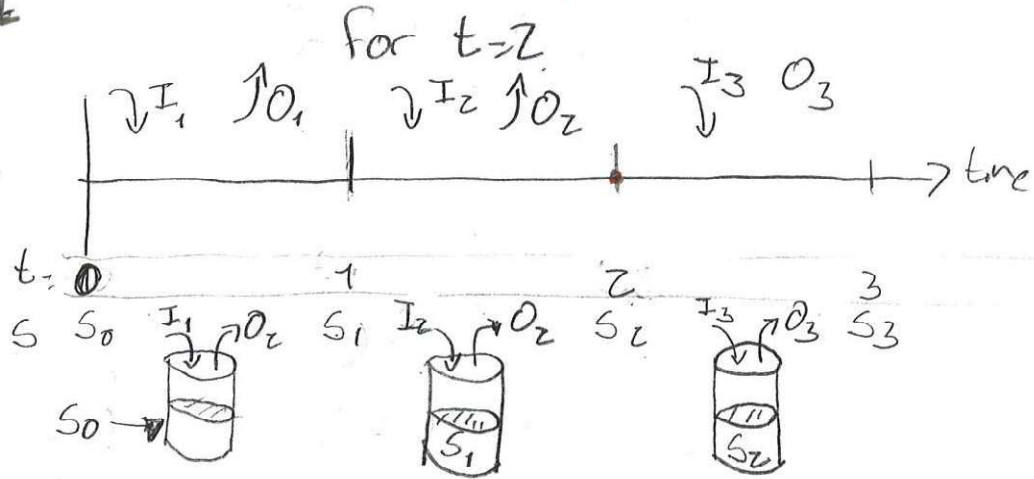
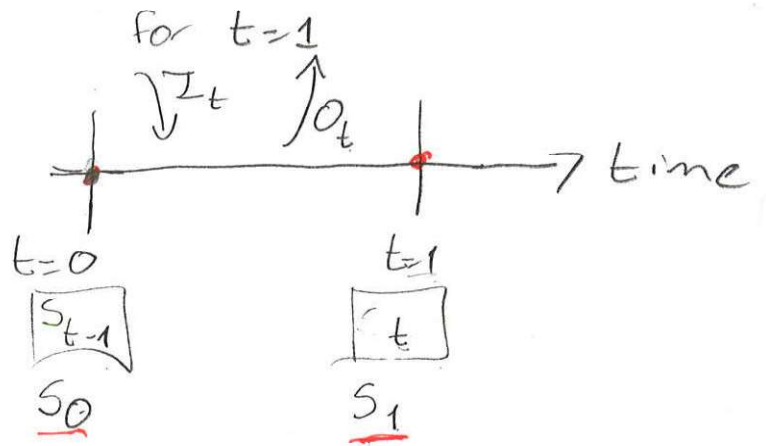
$$S_2 = S_1 + I_2 - O_2$$

For $t=3$

$$S_3 = S_2 + I_3 - O_3$$

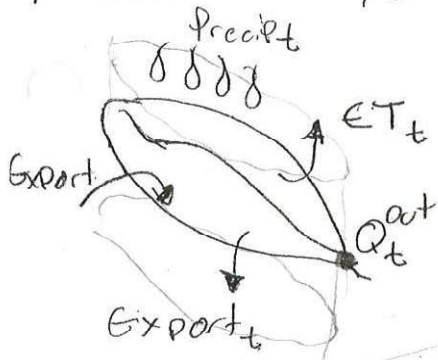
For $t=n$

$$S_n = S_{n-1} + I_n - O_n$$



| t | S_{t-1} | I_t | O_t | $S_t = S_{t-1} + I_t - O_t$ |
|-----|-----------|-------|-------|-----------------------------|
| 0 | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |

1) Natural System



$$\Delta S_t = I_t - O_t$$

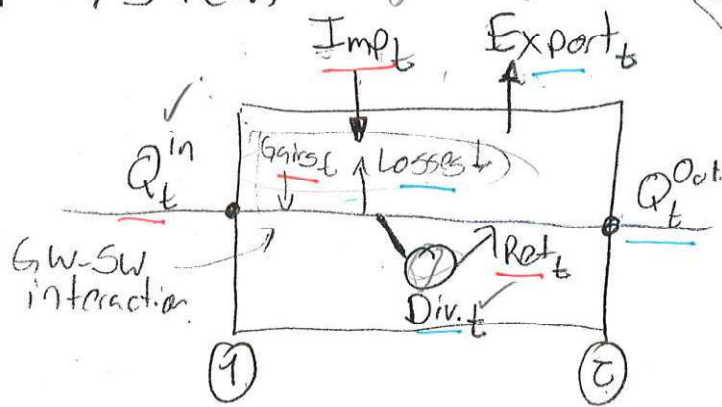
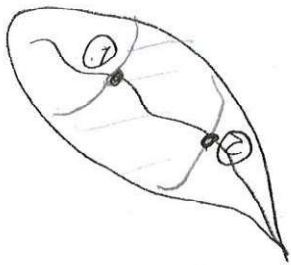
$$I_t = \text{Rain}_t + \text{Import}_t$$

$$O_t = \text{ET}_t + \text{Export}_t + Q_t^{\text{out}}$$

$$\Delta S_t = [\text{Rain}_t + \text{Import}_t] - [\text{ET}_t + \text{Export}_t + Q_t^{\text{out}}]$$

2) Surface Water System

For the exercise and exam



$$\Delta S_t = I_t - O_t$$

$$I_t = Q_t^{\text{in}} + \text{Import}_t + \text{Ret}_t + \text{Gains}_t$$

$$O_t = Q_t^{\text{out}} + \text{Export}_t + \text{Div}_t + \text{Losses}_t$$

No surface storage $\Delta S_t = 0$

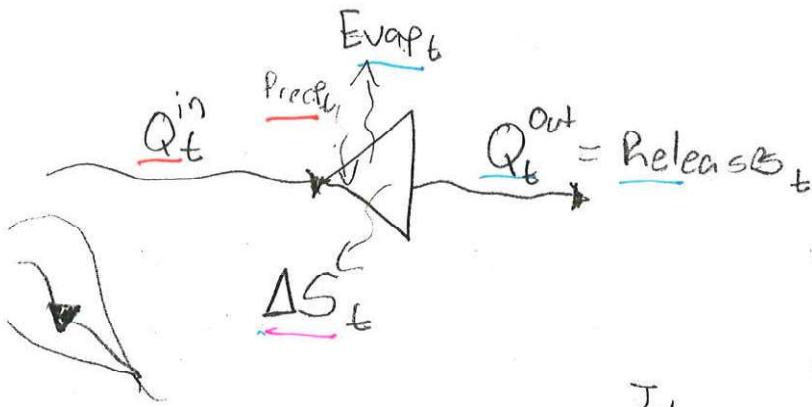
$$0 = I_t - O_t$$

$$I_t = O_t$$

$$Q_t^{\text{in}} + \text{Import}_t + \text{Ret}_t + \text{Gains}_t = Q_t^{\text{out}} + \text{Export}_t + \text{Div}_t + \text{Losses}_t$$

$$\text{Gains}_t - \text{Losses}_t = [Q_t^{\text{in}} + \text{Import}_t + \text{Ret}_t] - [Q_t^{\text{out}} + \text{Export}_t + \text{Div}_t]$$

3) Surface Water Reservoir.



$$\Delta S_t = I_t - O_t$$

$$\Delta S_t = S_t - S_{t-1}$$

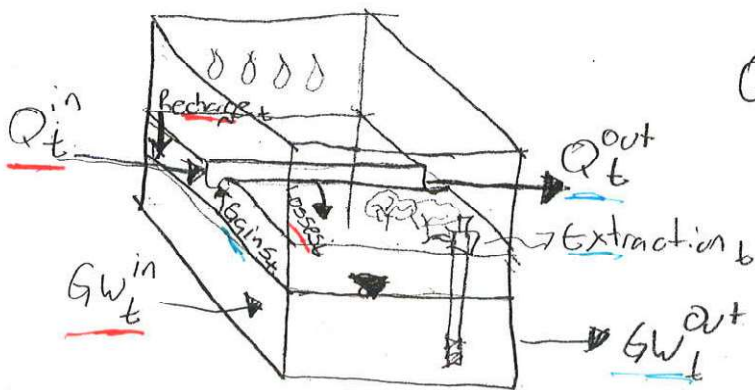
$$I_t = Q_t^{\text{in}} + \text{Precip}_t$$

$$O_t = Q_t^{\text{out}} + \text{Evap}_t$$

$$S_t - S_{t-1} = \left[Q_t^{\text{in}} + \text{Precip}_t \right] - \left[Q_t^{\text{out}} + \text{Evap}_t \right]$$

$$S_t = S_{t-1} + \left[Q_t^{\text{in}} + \text{Precip}_t \right] - \left[Q_t^{\text{out}} + \text{Evap}_t \right]$$

4) Surface - Groundwater system.



$$\Delta S_t = I_t - O_t$$

$$I_t = Q_t^{\text{in}} + \text{Recharge}_t + \text{GW}_t^{\text{in}} + \text{Losses}_t$$

$$O_t = Q_t^{\text{out}} + \text{Extraction}_t + \text{GW}_t^{\text{out}} + \text{Gainst}_t$$

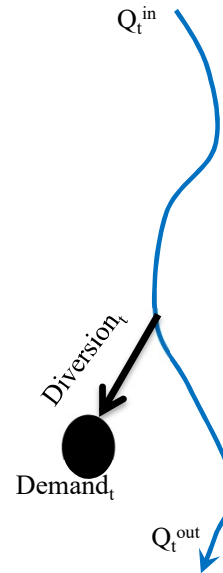
$$\Delta S_t = S_t - S_{t-1}$$

$$S_t - S_{t-1} = \left[Q_t^{\text{in}} + \text{Recharge}_t + \text{GW}_t^{\text{in}} + \text{Losses}_t \right] - \left[Q_t^{\text{out}} + \text{Extraction}_t + \text{GW}_t^{\text{out}} + \text{Gainst}_t \right]$$

$$\Delta S_t = I_t - O_t$$

$$\Delta S_t = 0, \text{ then } I_t = O_t$$

| | t | Q_t^{in} | Demand _t | Diversi _o n _t | Q_t^{out} |
|------|----|------------|---------------------|-------------------------------------|-------------|
| 2015 | 1 | 10 | 2 | | |
| 2015 | 2 | 9 | 2 | | |
| 2015 | 3 | 12 | 2 | | |
| 2015 | 4 | 6 | 6 | | |
| 2015 | 5 | 2 | 6 | | |
| 2015 | 6 | 1 | 6 | | |
| 2015 | 7 | 5 | 11 | | |
| 2015 | 8 | 0 | 11 | | |
| 2015 | 9 | 0 | 11 | | |
| 2015 | 10 | 0 | 5 | | |
| 2015 | 11 | 4 | 5 | | |
| 2015 | 12 | 7 | 5 | | |
| 2016 | 1 | 11 | 2 | | |
| 2016 | 2 | 8 | 2 | | |
| 2016 | 3 | 7 | 2 | | |
| 2016 | 4 | 7 | 6 | | |
| 2016 | 5 | 1 | 6 | | |
| 2016 | 6 | 0 | 6 | | |
| 2016 | 7 | 0 | 11 | | |
| 2016 | 8 | 0 | 11 | | |
| 2016 | 9 | 4 | 11 | | |
| 2016 | 10 | 3 | 5 | | |
| 2016 | 11 | 7 | 5 | | |
| 2016 | 12 | 11 | 5 | | |



Mass Balance

Unknown Variables: Q_t^{out} , $Diversi_0)n_t$

$$Q_t^{out} (Q_t^{in}, Diversi_0)n_t)$$

$$Diversi_0)n_t (Q_t^{in})$$

Diversi_0)n_t (Q_t^{in})

$Diversi_0)n_t = Demand_t$ if $Q_t^{in} \geq Demand_t$

$Diversi_0)n_t = Q_t^{in}$ if $Q_t^{in} < Demand_t$

Streamflow Out

$$Q_t^{out} = Q_t^{in} - Diversi_0)n_t$$

$$S_0 = 500 \quad \text{TAF}$$

$$\Delta S_t = I_t - O_t$$

$$S_t - S_{t-1} = I_t - O_t$$

$$S_t = S_{t-1} + I_t - O_t$$

| | t | S_{t-1} | Q_t^{in} | Demand _t | Q_t^{out} | $S_t = S_{t-1} + I_t - O_t$ |
|------|----|-----------|-------------------|---------------------|--------------------|-----------------------------|
| 2015 | 1 | | 10 | 2 | 2 | |
| 2015 | 2 | | 9 | 2 | 2 | |
| 2015 | 3 | | 12 | 2 | 2 | |
| 2015 | 4 | | 6 | 6 | 2 | |
| 2015 | 5 | | 2 | 6 | 2 | |
| 2015 | 6 | | 1 | 6 | 2 | |
| 2015 | 7 | | 5 | 11 | 2 | |
| 2015 | 8 | | 0 | 11 | 2 | |
| 2015 | 9 | | 0 | 11 | 2 | |
| 2015 | 10 | | 0 | 5 | 2 | |
| 2015 | 11 | | 4 | 5 | 2 | |
| 2015 | 12 | | 7 | 5 | 2 | |
| 2016 | 1 | | 11 | 2 | 2 | |
| 2016 | 2 | | 8 | 2 | 2 | |
| 2016 | 3 | | 7 | 2 | 2 | |
| 2016 | 4 | | 7 | 6 | 2 | |
| 2016 | 5 | | 1 | 6 | 2 | |
| 2016 | 6 | | 0 | 6 | 2 | |
| 2016 | 7 | | 0 | 11 | 2 | |
| 2016 | 8 | | 0 | 11 | 2 | |
| 2016 | 9 | | 4 | 11 | 2 | |
| 2016 | 10 | | 3 | 5 | 2 | |
| 2016 | 11 | | 7 | 5 | 2 | |
| 2016 | 12 | | 11 | 5 | 2 | |

