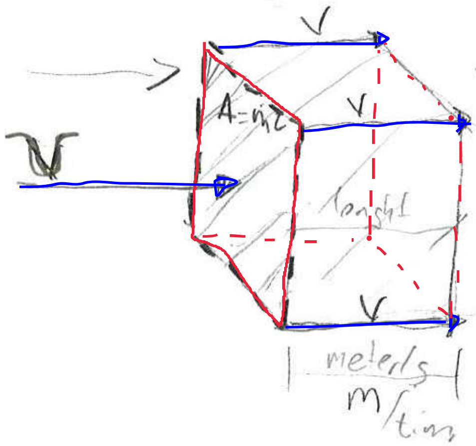
# 1) Continuity Equation




Flow — Velocity — Area

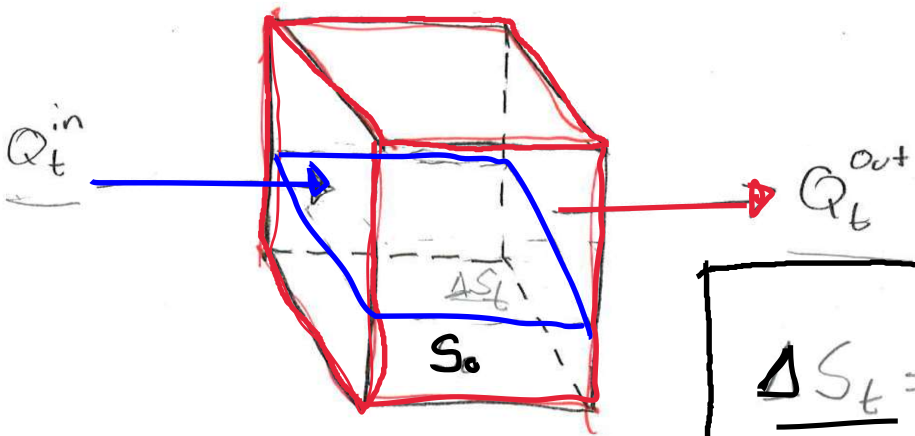
$$Q = V \cdot A$$

Volume/time: $\frac{m^3}{s} = \frac{m}{s} \cdot m^2$

$$\frac{ft^3}{s} = \frac{ft}{s} \cdot ft^2$$

at every second

## Control Volume




$$\Delta S_t = I_t - O_t$$

$$\Delta S_t = Q_t^{in} - Q_t^{out}$$

**Mass Balance**

$$\Delta S_t = I_t - O_t$$

$$S_t - S_{t-1} = I_t - O_t$$

$$S_t = S_{t-1} + I_t - O_t$$

For $t=1$

$$S_1 = S_0 + I_1 - O_1$$

For $t=2$

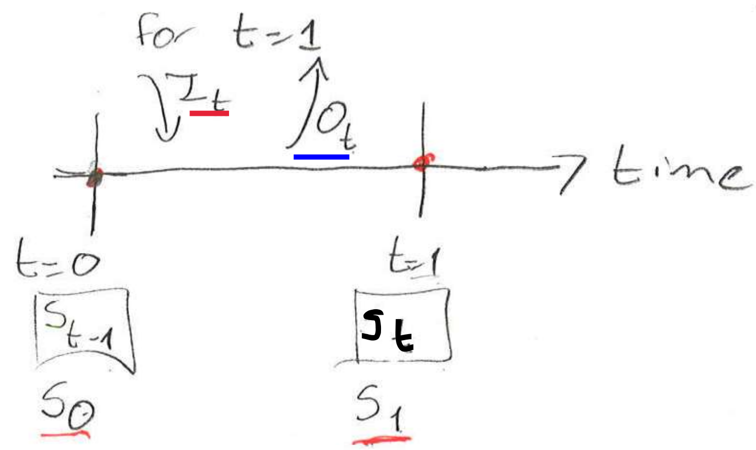
$$S_2 = S_1 + I_2 - O_2$$

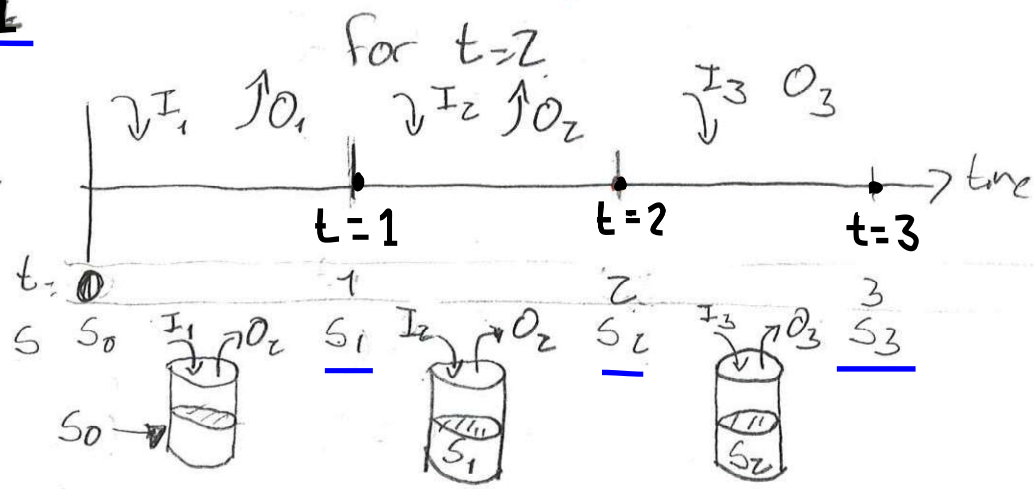
For $t=3$

$$S_3 = S_2 + I_3 - O_3$$

For $t=n$

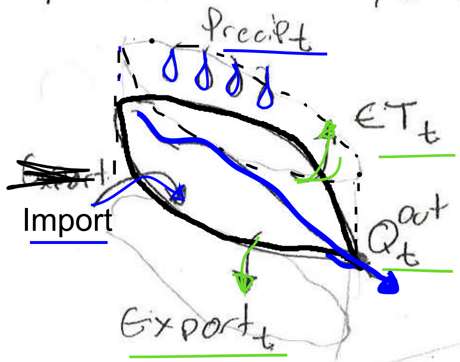
$$S_n = S_{n-1} + I_n - O_n$$




| $t$ | $S_{t-1}$ | $I_t$ | $O_t$ | $S_t = S_{t-1} + I_t - O_t$ |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |

## 1) Natural System




$$\Delta S_t = I_t - O_t$$

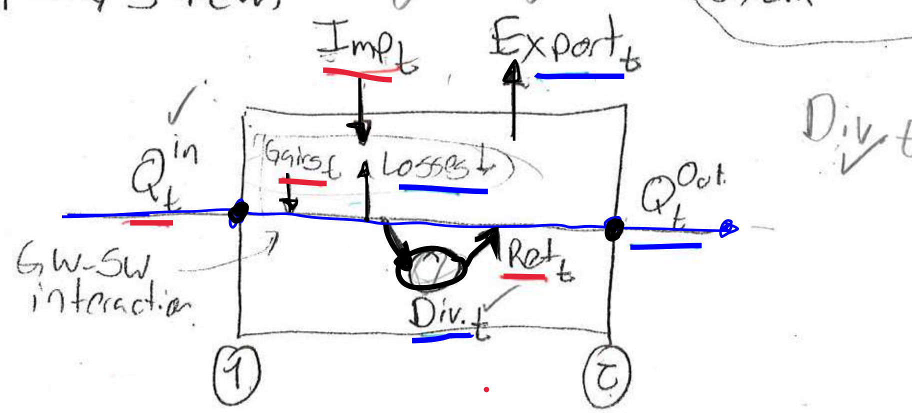
$$I_t = Rain_t + Import_t$$

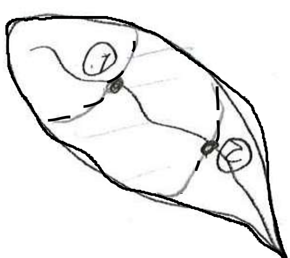
$$O_t = ET_t + Export_t + Q_t^{out}$$

$$\Delta S_t = \underbrace{[Rain_t + Import_t]}_{I_t} - \underbrace{[ET_t + Export_t + Q_t^{out}]}_{O_t}$$

## 2) Surface Water System

* For the exercise and Exam




$$\Delta S_t \to 0 = I_t - O_t ;$$

$$I_t = Q_t^{in} + Import_t + Ret_t + Gains_t$$

$$O_t = Q_t^{Out} + Export_t + Div._t + Losses_t$$

No surface Storage $\Delta S_t = 0$

$$0 = I_t - O_t$$

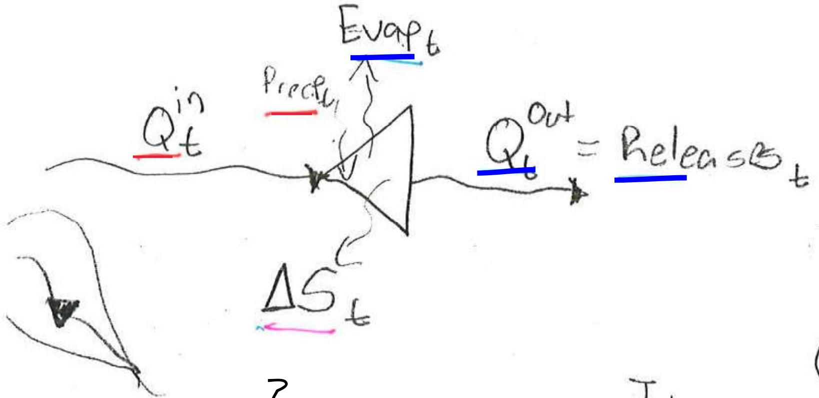
$$I_t = O_t$$

$$Q_t^{in} + Import_t + Ret_t + Gains_t = Q_t^{Out} + Export_t + Div._t + Losses_t$$

$$Gains_t - Losses_t = [Q_t^{in} + Import_t + Ret_t] -$$

$$[Q_t^{Out} + Export_t + Div_t]$$

3) Surface Water Reservoir.




$$\Delta S_t = I_t - O_t$$

$$\Delta S_t = S_t - S_{t-1}$$

$$I_t = Q_t^{in} + Precip_t$$

$$O_t = Q_t^{out} + Evap_t$$

$$S_t - S_{t-1} = \underbrace{\left[Q_t^{in} + Precip_t\right]}_{I_t} - \underbrace{\left[Q_t^{out} + Evap_t\right]}_{O_t}$$

$$S_t = S_{t-1} + \left[Q_t^{in} + Precip_t\right] - \left[Q_t^{out} + Evap_t\right]$$

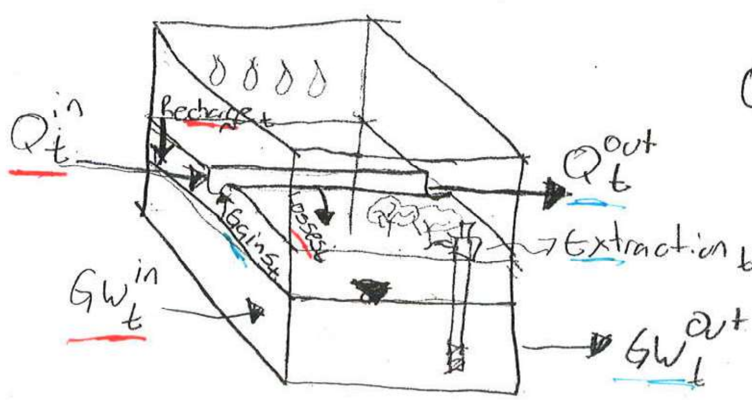
4.) Surface - Groundwater system.

$$\Delta S_t = I_t - O_t$$

$$I_t = Q_t^{in} + Recharge_t + GW_t^{In} + Losses_t$$

$$O_t = Q_t^{out} + Extraction_t + GW_t^{Out} + Gains_t$$

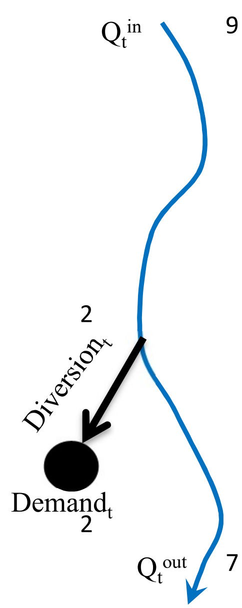
$$\Delta S_t = S_t - S_{t-1}$$




$$S_t - S_{t-1} = \underbrace{\left[Q_t^{in} + Recharge_t + GW_t^{In} + Losses_t\right]}_{I_t} - \underbrace{\left[Q_t^{Out} + Extraction_t + GW_t^{Out} + Gains_t\right]}_{O_t}$$

**$\Delta S_t = I_t - O_t$** **because $\Delta S_t = 0$, then $I_t = O_t$**

| | t | $Q^{in}_t$ | $Demand_t$ | $Diversion_t$ | $Q^{out}_t$ |
|---|---|---|---|---|---|
| 2015 | 1 | 10 | 2 | 2 | 8 |
| 2015 | 2 | 9 | 2 | 2 | 7 |
| 2015 | 3 | 12 | 2 | 2 | 10 |
| 2015 | 4 | 6 | 6 | 6 | 0 |
| 2015 | 5 | 2 | 6 | 2 | 0 |
| 2015 | 6 | 1 | 6 | 1 | 0 |
| 2015 | 7 | 5 | 11 | 5 | 0 |
| 2015 | 8 | 0 | 11 | 0 | 0 |
| 2015 | 9 | 0 | 11 | 0 | 0 |
| 2015 | 10 | 0 | 5 | 0 | 0 |
| 2015 | 11 | 4 | 5 | 4 | 0 |
| 2015 | 12 | 7 | 5 | 5 | 2 |
| 2016 | 1 | 11 | 2 | 2 | 9 |
| 2016 | 2 | 8 | 2 | 2 | 6 |
| 2016 | 3 | 7 | 2 | 2 | 5 |
| 2016 | 4 | 7 | 6 | 6 | 1 |
| 2016 | 5 | 1 | 6 | 1 | 0 |
| 2016 | 6 | 0 | 6 | 0 | 0 |
| 2016 | 7 | 0 | 11 | 0 | 0 |
| 2016 | 8 | 0 | 11 | 0 | 0 |
| 2016 | 9 | 4 | 11 | 4 | 0 |
| 2016 | 10 | 3 | 5 | 3 | 0 |
| 2016 | 11 | 7 | 5 | 5 | 2 |
| 2016 | 12 | 11 | 5 | 5 | 6 |




**Mass Balance**

Unknown Variables: $Q_t^{Out}$, $Diversion_t$

$Q_t^{Out}(Q_t^{in}, Diversion_t)$

$Diversion_t(Q_t^{in})$

**$Diversion_t(Q_t^{in})$**

$Diversion_t = Demand_t$ if $Q_t^{in} >= Demand_t$

$Diversion_t = Q_t^{in}$ if $Q_t^{in} < Demand_t$

**Streamflow Out**

$Q_t^{Out} = Q_t^{in} - Diversion_t$

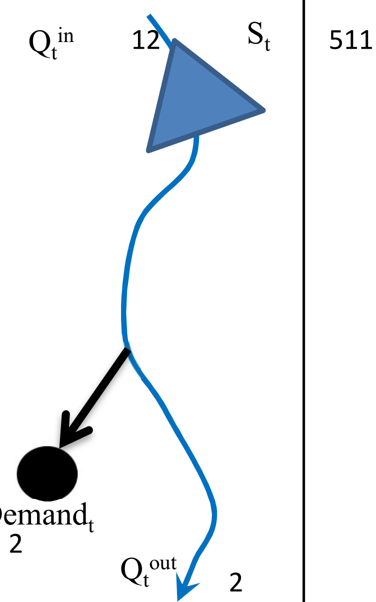
$S_0 =$ *500* TAF

$$\Delta S_t = I_t - O_t$$

$$S_t - S_{t-1} = I_t - O_t$$

$$S_t = S_{t-1} + I_t - O_t$$

| | t | $S_{t-1}$ | $Q^{in}_t$ | $Demand_t$ | $Q^{out}_t$ | um Outflow | $S_t = S_{t-1} + I_t - O_t$ |
|---|---|---|---|---|---|---|---|
| 2015 | 1 | 500 | 10 | 2 | 2 | 4 | 506 |
| 2015 | 2 | 506 | 9 | 2 | 2 | 4 | 511 |
| 2015 | 3 | 511 | 12 | 2 | 2 | 4 | 519 |
| 2015 | 4 | 519 | 6 | 6 | 2 | 8 | 517 |
| 2015 | 5 | 517 | 2 | 6 | 2 | 8 | 511 |
| 2015 | 6 | 511 | 1 | 6 | 2 | 8 | 504 |
| 2015 | 7 | 504 | 5 | 11 | 2 | 13 | 496 |
| 2015 | 8 | 496 | 0 | 11 | 2 | 13 | 483 |
| 2015 | 9 | 483 | 0 | 11 | 2 | 13 | 470 |
| 2015 | 10 | 470 | 0 | 5 | 2 | 7 | 463 |
| 2015 | 11 | 463 | 4 | 5 | 2 | 7 | 460 |
| 2015 | 12 | 460 | 7 | 5 | 2 | 7 | 460 |
| 2016 | 1 | 460 | 11 | 2 | 2 | 4 | 467 |
| 2016 | 2 | 467 | 8 | 2 | 2 | 4 | 471 |
| 2016 | 3 | 471 | 7 | 2 | 2 | 4 | 474 |
| 2016 | 4 | 474 | 7 | 6 | 2 | 8 | 473 |
| 2016 | 5 | 473 | 1 | 6 | 2 | 8 | 466 |
| 2016 | 6 | 466 | 0 | 6 | 2 | 8 | 458 |
| 2016 | 7 | 458 | 0 | 11 | 2 | 13 | 445 |
| 2016 | 8 | 445 | 0 | 11 | 2 | 13 | 432 |
| 2016 | 9 | 432 | 4 | 11 | 2 | 13 | 423 |
| 2016 | 10 | 423 | 3 | 5 | 2 | 7 | 419 |
| 2016 | 11 | 419 | 7 | 5 | 2 | 7 | 419 |
| 2016 | 12 | 419 | 11 | 5 | 2 | 7 | 423 |



**$\Delta S_t = I_t - O_t$**