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Improving Performance Criteria in the Water Resource Systems Based on Fuzzy Approach

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Abstract
Reliability, resilience, and vulnerability (RRV) have been widely used as the performance criteria of a water supply system in the studies conducted over the last three decades. This study attempts to modify the traditional method commonly applied to estimate these criteria using fuzzy logic thereby the performance criteria of the points with the threshold and intermediate values are more accurately estimated. Traditional methods (RRV-Fixed) of estimating these criteria are based on the fixed threshold values to represent the functionality of a water supply system, using a binary system to identify the periods a system fails to supply the water demands. The employment of this binary system may be taken into account as a weakness of the evaluating system, especially when water portion met is close to the threshold values. The present study develops a new method named RRV-Fuzzy, to ameliorate the weaknesses of the traditional RRV-Fixed estimating system.The method is designated as "*Fuzzy Performance Criteria*" built upon the traditional RRV formulae with improvements made to their structures using fuzzy membership functions. The efficiency of the proposed method is verified via implementation on two case studies including a theoretical and a real-world water basin. A comparison of the proposed RRV-Fuzzy and the traditional RRV-Fixed methods confirms the efficiency of the proposed method with regard to the improvements achieved in the relevant estimations, validating the new approach to be quite effective and practicable.

Keywords Reliability · Resilience · Vulnerability · Sustainability index · Water resource system · Fuzzy membership function

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1 Introduction

One of the most important challenges facing water resources planning and management is developing relevant criteria and indices to assess the health and performance of a water resource system (Ni et al. 2012). On the other hand, achieving a sustainable system is the main goal of Integrated Water Resources Management (IWRM) projects in the world (GWP 2000, 2003; Sandoval-Solis et al. 2011; Aydin 2014; Wang et al. 2019). Indicators are the most important components of sustainability analysis, and thus are required to be appropriate enough for each case of study (Recanati et al. 2017). Likewise, the concept of sustainability (International Union for Conservation of Nature (IUCN) 1980; WCED 1987) was first introduced in the 1980s. Then, Loucks (1997) developed" Sustainability Index" (SI), which was a combination of the common measures of Reliability, Resilience, and Vulnerability (RRV) first introduced by Hashimoto et al. (1982). SI is one of the most popular indices to express the overall performance of water supply systems that can be used to properly design and operate these systems (Karamouz et al. 2017; Yazdandoost et al. 2020). The sustainability index has been recently used in assessing renewable energy (Wang and Zhan 2019) and water-energy-food nexus (Wang et al. 2018). On the other hand, the SI concept is regarded as the most important indicator helping scientists detect the uncertainty of the systems behavior.

RRV criteria are risk-based measures widely used to quantify uncertainties in policy making and evaluate the performance of natural water resources and man-made water supply systems (Hashimoto 1980; El-Baroudy and Simonovic 2003; Sandoval-Solis et al. 2011; Safavi et al. 2016; Vieira et al. 2018; Ren et al. 2020). Either of the two stochastic (probabilistic) or fuzzy approaches may be used to assess the uncertainty in these systems (El-Baroudy and Simonovic 2006). The stochastic approach is used for those systems or models where randomness is a prevalent way to express uncertainty (Klir and Yuan 1995). Lack of knowledge and information is another source of uncertainty that should be considered in cases where mathematical procedures cannot quantify uncertainties. To account for uncertainties in performance criteria, the occurrence of a failure could be treated as a fuzzy event (Rehana and Mujumdar 2012). El-Baroudy and Simonovic (2003, 2006) provided a new fuzzy approach to conceptualize this type of uncertainty. Their research was focused on using fuzzy sets to quantify uncertainties resulting from the lack of knowledge. Kumari and Mujumdar (2017) developed fuzzy set-based measures to estimate fuzzy reliability, resilience, and vulnerability. Their research indicates that fuzzy performance measures present the realistic estimations in the reservoir operation policies. Other studies on the use of RRV concepts can be found in (Asefa et al. 2014; Mashhadi Ali et al. 2017, and Yazdandoost et al. 2020).

This study attempts to compare the current probabilistic RRV method that uses fixed thresholds, and hence is named RRV-Fixed method with a proposed method using the membership function concepts defined in the fuzzy approach to estimate RRV, and hence is named RRV-Fuzzy. The proposed method is to improve the traditional probabilistic RRV criteria, which is different from the previous studies (e.g., El-Baroudy and Simonovic 2003, 2006; Kumari and Mujumdar 2017). One of the weaknesses of the traditional methods is related to the threshold value, focusing on which may disrupt the process of distinguishing the failure states of the systems. This study proposes a new fuzzy approach to improve the probabilistic RRV estimations in the threshold values using a fuzzy membership function whose internal parameters are defined and tuned utilizing the viewpoints of the experts and/or the stakeholders. We examined four cases of the most well-known fuzzy membership functions and eventually favored one of them showing the best performance. Accordingly, RRV-

Fixed and RRV-Fuzzy criteria are calculated for two theoretical and real-world case studies. The Zayandehrud basin taken as the real-world case study is one of the most important watersheds in west-central Iran.

2 Study Area and Data

2.1 Case Study: The Zayandehrud River Basin

The Zayandehrud River basin is selected as a real case of study to assess the proposed approach. This river is about 350 km long and runs in a roughly west-east direction. It originates from the Zardkuh-Bakhtiari Mountains, Southwest of the Isfahan province, ending in the Gavkhuni Wetland, east of the Isfahan city (Murray-Rast et al. 2000). The Zayandehrud River (87%), the Pelasjan River (12%), and the Samandegan River (less than 1%) are the main surface water sources flowing to the Zayandehrud dam constructed in 1971 (Fig. 1). Kuhrang Tunnel No. 1 with a capacity of 340 MCM, Kuhrang Tunnel No. 2 with a capacity of 250 MCM, and Cheshme-Langan Tunnel with a capacity of 340 MCM divert water from the other basins located in southwest and west of the basin (Fig. 1). Groundwater accounts for another source of water supplying the water demands of the Zayandehrud basin. Figure 1 shows the boundaries of the sub-basins and aquifers. The municipal, environmental, industrial, and agricultural water deamnds in the Zayandehrud basin are supplied by the conjunctive use of surface and groundwater resources. The water demands in the basin supplied by the surface and groundwater resources as well as the portion supplied by each of these resources have been shown in Fig. 1. Although the Zayandehrud river is the major source of water in the basin, water users strongly rely on the groundwater resources to meet their water requirements, as well.

2.2 Zayandehrud Water Management Models

There are two water main models used to conduct the current and proposed water management practices to facilitate the water supply systems evaluation process to be carried out in the Zayandehrud basin: (1) ANFIS Model as a rainfall-runoff model and (2) the Water Evaluation and Planning System (WEAP) platform as a priority-based water allocation model to illustrate infrastructure, regional hydrology, and water allocation system on a monthly basis (Planning Model). The WEAP model used on a 21-year hydrologic period of analysis (Oct/1991-Sept/2011), which calculates the monthly water balance of inflows, changes in reservoirs and groundwater storage volumes, water supply allocated to water demands, and outflows. More detailed explanations about the water planning model and the process of the model calibration and validation are provided in Safavi et al. (2015). Figure 2 illustrates a scheme of the modeling steps and the methodology of this study.

2.3 Baseline Scenario Development

The baseline scenario was proposed for evaluating the water supply system performance under current water management policies in the basin. This scenario is developed under A1 conditions of climate change possibly occurring in the future and represents a water management scheme before any new policy is adopted and implemented for the near future (Oct. 2020

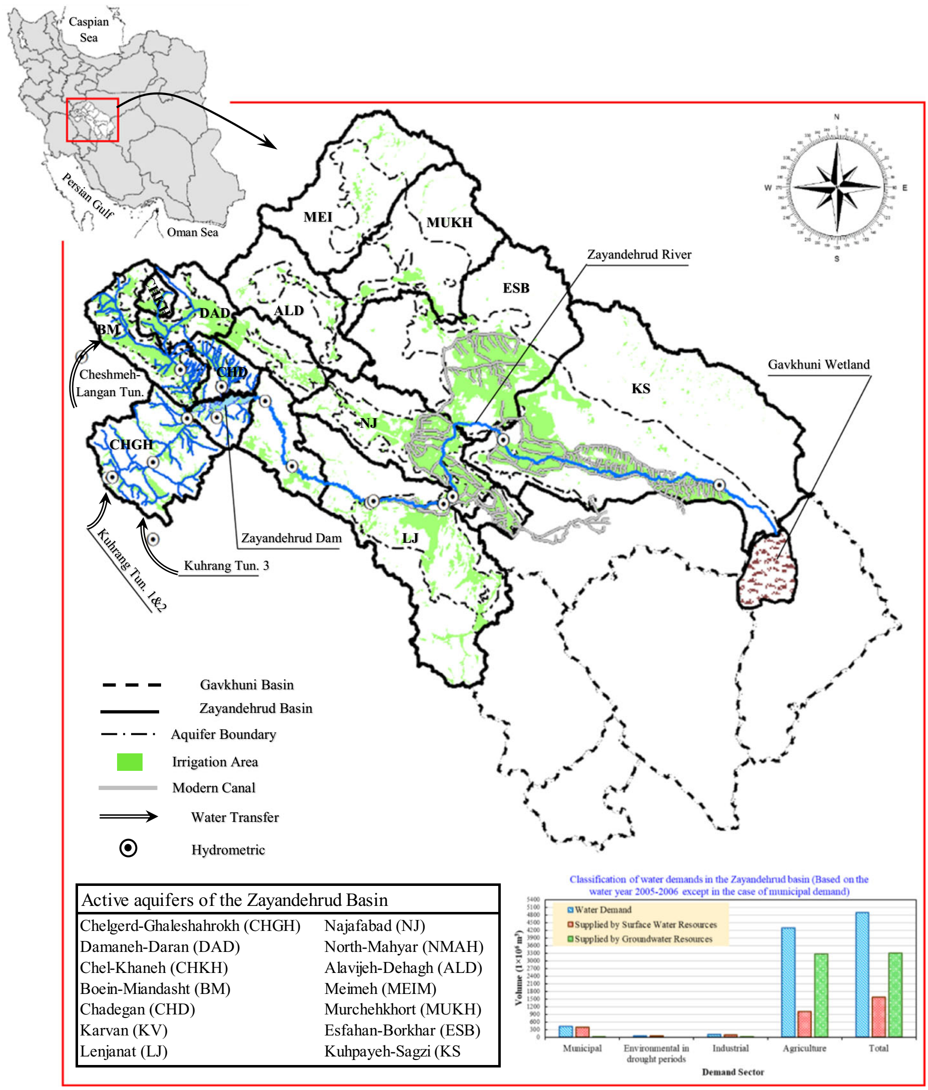

Fig. 1 Location of Gavkhuni basin in Iran; Gavkhuni and Zayandehrud basins, geography of the basins, aquifers, sub-basins, and classification of water demands

to Sep. 2025). Water supply systems performance in the near future was thus evaluated for every type of water demand using the water stored in all the aquifers and the reservoir under the current water management policies. RRV criteria and SI values were also estimated to evaluate the efficiency of the baseline scenario. Figures 3a and b present the results obtained on the water supplied in all the municipal, industrial, agricultural, and environmental sections as well as surface and groundwater resources in the Zayandehrud basin. The desirable aquifer and storage levels were determined based on the viewpoints of the local experts who expressed that the monthly conditions of the water year 2007, reported in Figs. 3b could be assumed as the normal and desirable conditions for the Zayandehrud Dam and aquifers. The detailed explanations of the Baseline scenario, its assumptions, climate change data, ANFIS model and

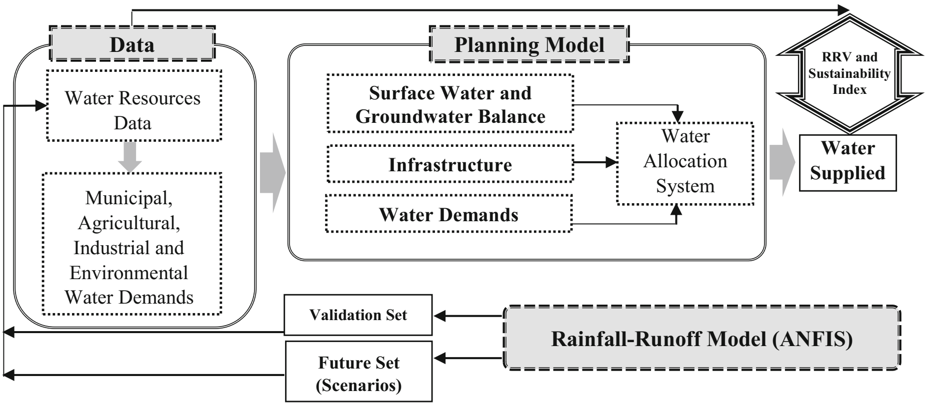

Fig. 2 The conceptual model for the study

its structure, and the the method estimating the river water recharging the aquifer can be found in Safavi et al. (2015).

Using the values obtained for water supplies and water resources versus the water demands and the desired volumes of the Zayandehrud dam and aquifers, performance criteria were estimated to evaluate the sustainability of the Zayandehrud basin under the baseline scenario.

3 Methodology

3.1 Traditional Method

3.1.1 Problem Statement

The essential purpose of the reservoir operation is to supply demands at the right time and volume, defined as the reliability criteria in time and volume. As defined by Hashimoto et al. (1982), reliability is the probability that the system will remain in a non-failure state at any given time; resilience is the ability of the system to return to a non-failure state after a failure has occurred; and vulnerability is the likely damage resulting from a failure event (Asefa et al. 2014; Karamouz et al. 2017; Vieira et al. 2018; Ren et al. 2020), providing a measurement of the potential damage caused when a system is falling into a state of failure (Hoque et al. 2012). These criteria are related to water demand ($Demand_t^j$) and water supply ($Supply_t^j$) for a pre-determined jth water user (Eq. 1) in four sectors of municipal, agricultural, environmental, and industrial. The performance criteria can also be related to the reservoir status ($Reserv_t^j$) and its desired status ($Desire_t^j$) for a pre-determined jth water resource representing an aquifer or a reservoir. The same equations are used for the performance criteria of both water users and water resources (Vigerstol 2002), and these equations are only required to replace $Demand_t^j$, $Reserv_t^j$, $Supply_t^j$ and $Desire_t^j$ to each other to facilitate calculating the relevant objective function. If deficit (D_t^j) is defined as (Loucks 1997):

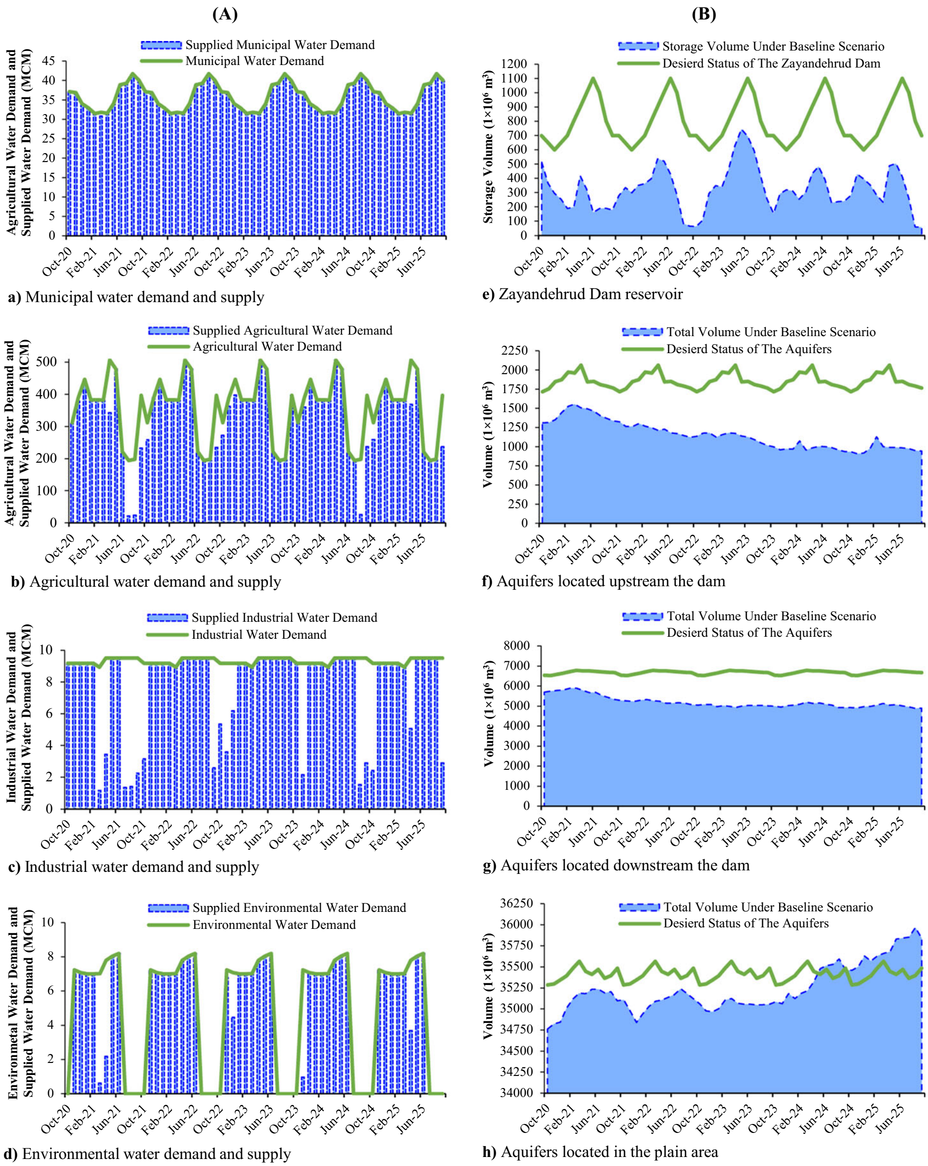

Fig. 3 **A** Total water demand and supply delivered under the baseline scenario. B The Zayandehrud water resources storage volumes under the baseline scenario and desired status

$$D_t^j = \begin{cases} Demand_t^j - Supply_t^j & ifDemand_t^j > Supply_t^j \\ 0 & ifDemand_t^j = Supply_t^j \end{cases} \quad (1)$$

The following equations illustrate the mathematical procedure for estimating the RRV: reliability in time (McMahon et al. 2006); reliability in volume (Hashimoto et al. 1982), resilience (Hashimoto et al. 1982; Moy et al. 1986); and vulnerability (Sandoval-Solis et al. 2011), respectively:

$$Rel_{time}{}^{j} = \frac{N_s}{T} \times 100\% \forall t = 1, 2, \ldots, T; 0 \leq Rel_{time}{}^{j} \leq 100\% \tag{2}$$

$$Rel_{Vol}{}^{j} = \frac{\sum_{t=1}^{T} Supply_t^j}{\sum_{t=1}^{T} Demand_t^j} \times 100\% = \frac{S}{D} \times 100 \quad 0 \leq Rel_{Vol}{}^{j} \leq 100\% \tag{3}$$

$$Res^{j} = \frac{N_{t=1}{}^{T}\left(D_{t+1}^j = 0 \,|\, D_t^j > 0\right)}{N_{t=1}{}^{T}\left(D_t^j > 0\right)} \times 100\% \forall t = 1, 2, \ldots, T; 0 \leq Res^{j} \leq 100\% \tag{4}$$

$$Vul^{j} = \frac{\sum_{t=1}^{T}\left(D_t^j | D_t^j > 0\right)}{\left[N_{t=1}{}^{T}\left(D_t^j > 0\right)\right] \frac{\sum_{t=1}^{T} Demand_t^j}{T}} \times 100\% \forall t = 1, 2, \ldots, T; 0 \leq Vul^{j} \leq 100\% \tag{5}$$

where N_s is the number of time steps in which the water demand was fully supplied, T is the total number of steps, and $N_{t=1}{}^{T}()$ is the number of time periods in which the system had the conditions stated in () (Ashofteh et al. 2015). S and D are the sum of water supplies and demands among all periods, respectively. The water distribution in the proper volume and at the proper time could guarantee the proper water resources management. This way to distribute the water resources in time and volume are interpreted by time-based reliability (Rel_{time}^{j}) and volumetric reliability (Rel_{Vol}^{j}), respectively. In a proper management, Rel_{time}^{j} and Rel_{Vol}^{j} should be close to each other, conceptually.

Depending on the purpose of each study, the vulnerability will have its own definition such as individual extent-vulnerability, cumulative extent-vulnerability, extent-vulnerability and conditional expected extent-vulnerability and so on (Loucks and Gladwell 1999).

The Water Resources Sustainability Index (SI^{j}) expressed as the geometric mean of RRV criteria (Loucks 1997) aggregated the performance criteria to result in a single index to facilitate comparisons among complex trade-offs; it is defined as follows (Sandoval-Solis et al. 2011; Karamouz et al. 2017):

$$SI^{j} = \left\{Rel_{time}{}^{j} \times Rel_{Vol}{}^{j} \times Res^{j} \times \left(1 - Vul^{j}\right)\right\}^{\frac{1}{4}} \tag{6}$$

Each of these performance criteria is expressed as a number between 0 and 1 (or 0% to 100%), in which a non-failure state (0) or a failure state (1) is defined through a binary logic classification (Zimmermann 2001). Regarding this classification in the reliability equation (Eq. 2), N_s is defined as the number of time steps in which the demand was fully supplied, meaning the number of time steps where $D_t^j = 0$.

As a theoretical example, assume a demand site with a demand of 50 units and seven different sets of water supplied (1 to 7), as seen in Table 1. Examples 1 to 5 were applied to RRV criteria to evaluate the system under the equal water supply volume in the year (356 units); and the examples 6 and 7 were used to show the system's resilience with severe and weak failures, respectively.

The difference between examples 1 and 2 is only lying in 0.1 unit of water supplied during the odd periods. In these examples, N_s values are revealed to be 7 and 0 for the examples 1 and 2, respectively, leading very different RRVs to be calculated for them.The reliability in time values (Rel^j_{time}) obtained in examples 1 and 2 are 53.85% and 0.0%, respectively and the reliability in volume values (Rel^j_{Vol}) are almost equal for these two examples (54.7%). Furthermore, the resilience values are achieved to be 100% and 0.0%; and the vulnerability values obtained from Eq. 5 are calculated to be 98% and 45.25%. The SI values for examples 1 and 2 obtained from Eq. 7 would be 27.71% and 0.0%, while the water supply system performances for these two examples are very similar, and one could find out that they are exactly the same. Examples 3 and 4 are provided to assess the new approach proposed in this study and compare the proposal with previous methods under intermediate and threshold conditions. In these examples, the supply values are more than half of demand in odd periods and less than half of demand in even periods. As a result, the reliability in time and reliability in volume for example 3 must be closer than those in example 4. On the contrary, the resilience for example 4 must be greater than that in example 3. However, the reliability in time and resilience values are 0%, obtained from Eqs. 2 and 4. Likewise, in example 5 demands are evenly supplied by available water. Thus, the Rel^j_{time} and Rel^j_{Vol} values should be close together, conceptually; while these values are obtained to be 0% and 54.76% from Eqs. 2 and 3, respectively. Likewise, the SI index was obtained to be 0% for these three examples (3 to 5). In example 6, the system has a great failure in even periods while in example 7, the failure is more tolerable. As a result, the resilience values for these two examples must be different, suggesting a more value for example 6 and a less value for example 7), whereas it is obtained to be 33.33% for both examples. The reason for this functionality of the water supply system in these exmaples is hidden in the binary system variables D^j_t (Eq. 1), N_s (Eq. 2), and $N_{t=1}^{T}\left(D^j_t > 0\right)$ (Eqs. 4 and 5). Therefore, a function should be developed to estimate the desirability value as an output varying from 0 to 1. One of the well-known techniques to achieve this goal is using fuzzy Membership Functions (MFs) (Klir and Yuan 1995; Zadeh 1997; Ross 2009).

3.2 Fuzzy Method

In ordinary set classification, the binary system makes an object either belonging to a set or not not belonging to that set. In this system, fuzzy sets represent a set of ordered pairs of elements for an object. The MF is the crucial component of the fuzzy set theory. A membership function maps a domain or space element to the unit interval [0, 1] (Pedrycz and Gomide 1998). Fuzzy MFs may have different shapes including continuous or discrete functions, depending on the context in which they are used (El-Baroudy and Simonovic 2003).

3.2.1 MF Definition

Gaussian, exponential, trapezoidal-shaped, sigmoidal, generalized bell-shaped, or triangular-shaped functions are well known fuzzy MFs used in many fields of research (Pedrycz and Gomide 1998; Zimmermann 2001). It is noteworthy that the defining a certain type of a membership function depends on the modeling conditions, the study area, the stakeholders' comments, and the experts' experiences. Considering all these factors in evaluating water supply performance criteria in the selection of the proper MF is one of the advantages and innovations of the proposed method. An appropriate and brief way to create an MF is to determine its parameters and then express the MF mathematically. In other words, the

Table 1 Demand and seven sets of water supply delivered

Period		1	2	3	4	5	6	7	8	9	10	11	12	13
Demand (unit)		**50**	**50**	**50**	**50**	**50**	**50**	**50**	**50**	**50**	**50**	**50**	**50**	**50**
Example 1	Water supplied 1	50.00	1.00	50.00	1.00	50.00	1.00	50.00	1.00	50.00	1.00	50.00	1.00	50.00
Example 2	Water supplied 2	49.90	1.1	49.90	1.1	49.90	1.1	49.90	1.1	49.90	1.1	49.90	1.1	49.90
Example 3	Water supplied 3	35.00	18.50	35.00	18.5	35.00	18.5	35.00	18.5	35.00	18.5	35.00	18.5	35.00
Example 4	Water supplied 4	42.30	10.00	42.30	10.00	42.30	10.00	42.30	10.00	42.30	10.00	42.30	10.00	42.30
Example 5	Water supplied 5	27.38	27.38	27.38	27.38	27.38	27.38	27.38	27.38	27.38	27.38	27.38	27.38	27.38
Example 6	Water supplied 6	50.00	1.00	2.00	49.00	2.00	50.00	30.00	49.00	1.00	50.00	1.00	5.00	50.00
Example 7	Water supplied 7	50.00	35.00	45.00	49.00	30.00	50.00	45.00	48.00	20.00	50.00	30.00	25.00	50.00

procedure of tuning an MF parameters is a primary and essential step to carry out prior to mathematically present that MF. Meanwhile, the process of the parameter setting of the MFs with strongly irregular shapes may not be so quick if not impossible in practice. (Mahmoud 2018). Moreover, linear MFs with Similar performances may be not so appropriate to use in every case. In this study, four instances of the MFs including pseudo-exponential, bell-shaped, sigmoidal, and gaussian MFs are competiting so that the best-performing and the lowest computationally expensive one is revealed as the proposed method. Because these MFs are nonlinear, smooth, and non-zero at all points, they have been widely used in many studies (Sumathi and Paneerselvam 2010; Mahmoud 2018).

Bell-Shaped MF The bell-shaped function is determined by three parameters, a, b and c; and defined as fellows:

$$\mu(x) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}; \begin{cases} 0 \le x \le 100\% \\ Otherwise \\ \mu(x) = 1 \end{cases} \tag{7}$$

where parameters a and c denote the half width and the center of the bell function, respectively; and b denotes and controls the slop at crossover point. Thus, any vector of (a, b, c) can define a unique bell-MF (Math Works 2020). The significant feature of the bell function is its nature to be symmetric, non-zero, and smooth at all points (Mahmoud 2018). Well tuning the three parameters of this function is the major problem facing this function, making it difficult to use in practice.

Pseudo-Exponential MF This function benefits from an ability to convert its output to a linear function even though several operators are used in its formula, which it's a major advantage compared to the other well-known MFs. Another benefit of an exponential function is that the linear MF is not an appropriate representative under the most of functional conditions, entailing this function to use the nonlinear MFs features(Li and Lee 1991). An exponential MF is generated based on two parameters, k, and m, as follows:

$$\mu(x) = \frac{1}{1 + k(x-m)^2}; \begin{cases} 0 \le x \le 100\% \\ Otherwise \\ \mu(x) = 1 \end{cases} \tag{8}$$

where k and m controls the slope at crossover point and denotes the center of the function, respectively.

Sigmoidal MF A sigmoidal MF is used in many applications, which is naturally open to the left or right. Sigmoidal MF, such as exponential MF is determined by two parameters, c and n. The parameter c is the center of the Sigmoidal MF varying between −1 to +1. The sign of the parameter n characterizes the extent of the sigmoidal MF. Hence, the parameter n is suitable for illustrating implications of Linguistic expressions like small, very small, large, very large, and less or more (Mahmoud 2018). The formula of this function is as follows:

$$\mu(x) = \frac{1}{1 + exp[-n(x-c)]}; \begin{cases} 0 \le x \le 100\% \\ Otherwise \\ \mu(x) = 1 \end{cases} \tag{9}$$

Gaussian MF Gaussian MF can be determined by two parameters as follows:

$$\mu(x) = exp\left[\frac{-1}{2}\left(\frac{x-c}{\sigma}\right)^2\right]; \begin{cases} 0 \le x \le 100\% \\ Otherwise \\ \mu(x) = 1 \end{cases} \tag{10}$$

where c and σ denote the center and the width of MF, respectively. These functions' curves are non-zero and smooth at every point, making these MFs quite favorite for application in fuzzy systems (Mahmoud 2018). According to the literature, Trapezoidal, Gaussian, and in some instances, triangular-shaped are the best and most well-known fuzzy MFs, among which the Gaussian MF is only used when nonlinear conditions appear (Wu 2012).

Fuzzy-Based RRV (RRV-Fuzzy) Depending on the sensitivity or priority of demands/sources to the water supply/their status, the MF could be changed by well setting its parameters, although the complexity of MFs and their parameters must be considered. Thus, the degree of desirability or undesirability could be adjusted for any demand or water resource, either wholly or independently. To fulfill the purposes of this study and based on the trade-offs among the 12 major stakeholders, decision-makers, and experts, MFs are compromised as the utility function for the Zayandehrud basin to estimate D_t^j in the fuzzy system (denoted by $\mu^j(x_t)$) which varies from 0 to 1. Based on the definition of RRV and their previously-mentioned formulae, the following new formulae are developed within the framework of the fuzzy approach to remove their weaknesses:

$$Rel_{time}^j = \frac{\sum_t \mu^j(x_t)}{T} \times 100\%; 0 \le Rel_{time}^j \le 100\% \tag{11}$$

$$Vul^j = \frac{\sum_t \left(1 - \mu^j(x_t)\right)}{T} \times 100\%; 0 \le Vul^j \le 100\% \tag{12}$$

For resilience, if resilience of jth user in time t regarding previous time (R_t^j) is defined as:

$$R_t^j = \begin{cases} 0 & if \quad \mu^j(x_t) < \mu^j(x_{t-1}) \\ \mu^j(x_{t-1}) & if \quad \mu^j(x_t) - \mu^j(x_{t-1}) < \alpha \\ \mu^j(x_t) - \mu^j(x_{t-1}) & O.W. \end{cases} \tag{13}$$

The following equation will then illustrate the mathematical procedure for estimating the system resilience:

$$\mathrm{Res}^j = \frac{\sum_t R_t^j}{N_{t=1}{}^T \left(R_t^j > 0\right)} \times 100\%; 0 \le \mathrm{Res}^j \le 100\% \tag{14}$$

where x_t denotes either the percentage of demand supplied ($\frac{S_t}{D_t}$) or the water resource status with respect to the desired status in time period t. $\mu^j(x_t)$ is the utility or desirability as estimated, and $1 - \mu^j(x_t)$ is a complement to $\mu^j(x_t)$ which represents an unsatisfactory grade. The parameter α is called "*System Resilience Significant Level,*" and it can change between 0 and 1 ($0 < \alpha < 1$). Indeed, the more the importance of the demand supplied, the closer the α to 0. This is a condition prescribed when a special type of the water demand such as the municipal water demand is focused to be supplied in practice. In contrast, this parameter is close to 1 for the demand with lower importance. In this study for demand sites, we assume α to be 0.1, and for water resources it is assumed to be 0.05. In these equations, j is the water users or water resources counter. Here, reliability in time is defined as the average desirability/utility during the simulation period or the period under investigation. Eq. 13 is a constrained equation to quantify resilience defined as the summation of the portion of the desirability increased from a certain time step to the next time step, divided by the total undesirable events. Vulnerability is quantified as the average undesirability during the simulation period or the period under investigation. It is noteworthy that Eqs. 11, 12, and 14 are developed based on Eqs. 1, 2, 3, and 5 and the concepts therein.

Parameters Estimation The parameters c and m (center of MFs) must either be characterized by experts, or it receives the most desirable value of the utility, i.e., 1. One the one hand, the utility is equal to 1 when $x = 1$ (100%) as x varies between 0 and 1 to reach the peak point of th MFs, and on the other hand, the utility is equal to 1 when $x = c$. As a result, c or m is equal to 1 (100%). The parameter a in bell-shaped is determined based on the expert's opinion, too, such that the more the undesirability in demand causes discontent, legally, the closer the a value to 0. Other parameters (b, k, n, and σ) are determined according to the theory developed in this study expressing the difference between Rel^j_{time} and Rel^j_{Vol} indicates that the available volume of water is allocated at inappropriate times. Hence, if water management is correctly conducted i.e., the accessible water volume assigned equal ratio at the different time steps, we will have follows:

$$x_1 = x_2 = x_3 = \ldots = x_t = \frac{S_t}{D_t} \times 100; \quad t = 1, 2, 3, \ldots, T \tag{15}$$

and, the desirability of the water demands portion met is defined by MFs as follows:

$$\mu(x_1) = \mu(x_2) = \mu(x_3) = \ldots = \mu(x_t) = \mu\left(\frac{S_t}{D_t} \times 100\right); \quad t = 1, 2, 3, \ldots, T \tag{16}$$

Regarding the above statement, in a right management, we should be have:

$$Rel_{time} = Rel_{Vol} \xrightarrow{Eq.\ 3,\quad Eq.\ 12\ and\ Eq.16} \mu(x_t) = \frac{S}{D} \tag{17}$$

where Rel_{time} can be any fuzzy MF that were characterized before (bell-shaped, exponential, sigmoidal, and Gaussian MFs), and Rel_{Vol} has constant value based on Eq. 3. Accordingly, by solving Eq. 17, the parameters b, k, n, and σ, for each MF could be estimated as follows:

$$Bell\text{-}shaped: b = \frac{1}{2}\left(\frac{ln\left(\dfrac{\sum_{t=1}^{T} Demand_t^j}{\sum_{t=1}^{T} Supply_t^j} - 1\right)}{ln\left|\dfrac{1}{a}\left[\left(\dfrac{\sum_{t=1}^{T} Supply_t^j}{\sum_{t=1}^{T} Demand_t^j}\right) - c\right]\right|}\right) \tag{18}$$

$$Pseudo\text{-}Exponential : k = \frac{\left(\frac{\sum_{t=1}^{T} Demand_t^j}{\sum_{t=1}^{T} Supply_t^j}\right)-1}{\left[\left(\frac{\sum_{t=1}^{T} Supply_t^j}{\sum_{t=1}^{T} Demand_t^j}\right)-m\right]^2} \tag{19}$$

$$Sigmoidal : a = \frac{-ln\left[\left(\frac{\sum_{t=1}^{T} Demand_t^j}{\sum_{t=1}^{T} Supply_t^j}\right)-1\right]}{\left(\frac{\sum_{t=1}^{T} Supply_t^j}{\sum_{t=1}^{T} Demand_t^j}\right)-c} \tag{20}$$

$$Gaussian : \sigma = \frac{\left(\frac{\sum_{t=1}^{T} Supply_t^j}{\sum_{t=1}^{T} Demand_t^j}\right)-c}{\left[-2ln\left(\frac{\sum_{t=1}^{T} Supply_t^j}{\sum_{t=1}^{T} Demand_t^j}\right)\right]^{\frac{1}{2}}} \tag{21}$$

The Best MF for RRV Estimating The selection of MFs in the fuzzy approach regarding their complexity and computational time is an essential step. On the other hand, the MFs shape is significant when assessing the system performance. For this purpose, the best system with the highest efficiency is to be achieved by frequently changing the MFs type or their internal parameters.

Bell-shaped is difficult to tune its three parameters. Moreover, the appropriateness of the shape of this MF may not be readily certified, suggesting this MF improper to use to delineate the water supply performance criteria. Furthermore, the parameters a and c in this function can be assigned different quantities and one may be in trouble to properly identify them. On the other hand, if the ratio of the water supply to the water demand is equal to 50%, for instance, or accordingly when if $a \geq x - c$, the bell function cannot generate different utility values for its different independent variables, indicating this function ineffective to calculate the performance criteria. Also, pseudo-exponential MF is of the same disadvantages as thebell MF shows, mainly as a result of having complexity in parameter m which can make the definition of an appropriate shape for this function very difficult.The significant point about exponential MF is that when Rel_{time}^j is equal to Rel_{Vol}^j then all the water supply to water demands ratios are equal, as illustrated in example 5, and thus the parameter m is equal to the water supply to water demand ratio at each time step, as can be inferred from Eq. 8.

The sigmoidal MF is more appropriate as compared to two MFs mentioned above. In this function, when $x = c$, the parameter a does not receive a real number (i.e., when the water supply is approximately equal to 50% of the water demand). Therefore, the membership function cannot be defined for this point, while having excellent performance in another points on the function.

Besides, the results of the theoretical examples in Problem Statement for the different scenarios justify Gaussian MF as the favorable MF. Therefore Gaussian MF is selected as the best fuzzy membership function, since the only parameter in this function is c that can be determined by experts. Here, to validate this methodology (RRV-Fuzzy), the results of the Gaussian MF on the theoretical example defined in "Problem Statement Section" are analyzed. Table 2 shows the RRV results obtained for the performance criteria and sustainability index estimated by the traditional method (RRV-fixed) compared with those obtained from the new approach proposed in this study (RRV-Fuzzy).

The results generated by the new approach show that RRV and SI are similar for examples 1 and 2, and are closer to the system's performance. Also, the new approach has a good performance in the proximity of the threshold conditions, whereas the previous method is not able to evaluate these conditions successfully, as is clear in the results presented by the RRV-Fixed and RRV-Fuzzy methods for the examples 2 to 5 in Table 2. In addition, examples 6 and 7 have been applied to assess the desirable and undesirable failure of system resilience. The results indicate that resilience in the new method presents more sensible and tangible values than those presented by the traditional method. Similarly, resilience has an equal value of 33.33% in the RRV-Fixed method compared to 59.78% and 74.91%, estimated by the RRV-Fuzzy approach. Interestingly, in Example 7, if the supply values of 50 are turned into 49.9, the reliability, resilience, and then SI in the RRV-Fixed method receive the zero values, while these values in RRV-Fuzzy do not experience significant variations. The new approach's performance is also evaluated for the baseline scenario in the Zayandehrud basin as a real-world example and compared with that of the traditional method.

Table 2 Comparison of performance criteria and sustainability indices in the7 examples provided in Table 1 using the previous and the new approaches

Performance Criteria (%)	Example 1	Example 2	Example 3	Example 4	Example 5	Example 6	Example 7
RRV-Fixed							
Reliability (Time)	53.85	0.00	0.00	0.00	0.00	30.77	30.77
Reliability (Volume)	54.77	54.75	54.77	54.78	54.76	52.31	81.08
Resilience	100	0.00	0.00	0.00	0.00	33.33	33.33
Vulnerability	98.00	45.25	45.23	45.22	45.24	68.89	27.33
*Sustainability Index	**27.71**	**0.00**	**0.00**	**0.00**	**0.00**	**35.94**	**49.58**
RRV-Fuzzy							
Reliability (Time)	56.58	56.61	55.67	57.23	54.76	54.39	73.85
Reliability (Volume)	54.77	54.75	54.77	54.78	54.76	52.31	81.08
Resilience	94.08	94.00	45.63	78.06	54.76	59.78	74.91
Vulnerability	43.42	43.39	44.33	42.77	45.24	45.61	26.15
*Sustainability Index	**63.73**	**63.73**	**52.76**	**61.18**	**54.76**	**55.15**	**75.87**

4 Results and Discussion

The RRV and SI performance criteria were estimated using the Zayandehrud WEAP model considering four major demanding sectors including municipal, agricultural, industrial, and environmental demands and fourteen major water resources including the surface water resources supplied by the Zayandehrud reservoir as well as the groundwater resources stored in the aquifers of the study area under the baseline scenario (Fig. 3a and b). Table 3a shows the performance criteria estimated using the RRV-Fixed and RRV-Fuzzy approaches. Vulnerability in the traditional method was estimated using Eq. 5.

It is clear from Fig. 3a that the system exhibits a very good performance in supplying water for the municipal demand; however, the RRV and SI values calculated by the RRV-Fixed approach (Table 3a) fail to reflect this performance in a satisfactory manner. SI is estimated to be 72.36%, whereas Fig. 3a indicates the system sustainability to be promising for supplying the municipal water demands. The major difference between the results presented by two examined methods is because the binary system variable of the RRV-fixed method considers deficits as a full-failure, although the deficit is quite small. The proposed RRV-Fuzzy approach estimates these criteria more satisfactarily mainly as a result of defining the fuzzy MFs utilizing the knowledge and experiences of the users and the authorities to hold a real and commensurate relation between the recorded deficits and the degrees of undesirability. Results of the new approach show that the reliability and resilience of the system estimated for the municipal

Table 3 Performance of (a) water supply and (b) water resources under the baseline scenario

(a) α = 0.1				
Performance Criteria (%)	Demands			
	Municipal	Agricultural	Industrial	Environmental
RRV-Fixed				
Reliability (Time)	73.33	66.67	73.33	90.00
Reliability (Volume)	99.93	91.90	81.77	91.44
Resilience	37.50	40.00	37.50	66.67
Vulnerability	0.25	24.29	68.38	85.60
*Sustainability Index	**72.36**	**65.63**	**51.64**	**53.02**
RRV-Fuzzy				
Reliability (Time)	98.74	85.04	76.17	91.97
Reliability (Volume)	99.93	91.90	81.77	91.44
Resilience	99.42	83.45	85.57	97.73
Vulnerability	1.26	14.96	23.83	8.03
*Sustainability Index	**99.21**	**86.30**	**79.82**	**93.24**
(b) α = 0.05				
Performance Criteria (%)	Water Resources			
	Surface Water Resource	Groundwater Resources (Aquifers)		
	Zayandehrud Dam	Upstream	Downstream	Plain Area
RRV-Fixed				
Reliability (Time)	0.00	0.00	33.06	23.89
Reliability (Volume)	40.77	55.12	81.22	96.53
Resilience	0.00	0.00	33.33	1.96
Vulnerability	59.23	44.88	18.88	3.56
*Sustainability Index	**0.00**	**0.00**	**51.91**	**25.70**
RRV-Fuzzy				
Reliability (Time)	43.15	55.65	80.83	96.27
Reliability (Volume)	40.77	55.12	81.22	96.53
Resilience	18.79	42.76	73.56	96.18
Vulnerability	56.85	44.35	19.17	3.73
*Sustainability Index	**34.5**	**51.98**	**79.05**	**96.32**

water demands is very high, while the vulnerability is very low, resulting in a sustainability of 99.21%. These results indicate that the new approach can evaluate the system's performance in a more proper and plausible manner.

For the remainder of the demands, the RRV values calculated based upon the RRV-Fuzzy method are slightly higher than those obtained by the traditional RRV-Fixed method. These results show the new approach has also a good performance in cases with intermediate conditions.

Table 3b shows the results obtained from evaluating the performance criteria of surface and groundwater resources by the two approaches under the baseline scenario. The reliability and resilience of the Zayandehrud reservoir and the aquifers located upstream this reservoir are estimated by the traditional RRV-Fixed method to be zero; therefore, SI would be zero. However, not only does the RRV-Fuzzy method not yield zero values for these indices,but generates far different outputs as the indices calculated for the performence of the aquifers located in the plain area, as compared to the RRV-Fixed method. As shown in Fig. 3h, the desired storage volume of these aquifers is about 35,400 MCM, and the volume of these aquifers calculated during the simulation period is close to the desired status. Thus, the performance of these aquifers estimated using the RRV-Fuzzy method does not report the sustainability of both water supply and water resources as low as that estimated by the RRV-Fixed method. The RRV-fuzzy method identified the performance criteria for the threshold values of the groundwater volume to be close to the desired status. It also estimated more reliable values, especially when knowing the real performance of these aquifers indicates that the sustainability index amounted to 96.32% is more plausible than that estimated to be 25.70%.

In general, the RRV-Fuzzy performance criteria reported in Table 3b indicate that the sustainability of the Zayandehrud water resources will be at inadequate, undesirable, and unacceptable level. Low reliability and resilience with the high vulnerability show that the Zayandehrud reservoir and the aquifers located upstream the reservoir will experience very acute conditions in the future. Unlike the predictions made by the traditional method, the new approach indicates that the reliability of the aquifers located downstream the reservoir and extended in the plain area will be at very good level.

While the estimates obtained from the RRV-Fixed method show that all the water supplies and the water resources in the Zayandehrud basin will be in critical conditions, the fuzzy performance criteria indicate that a certain number of them such as the municipal demand and the aquifers located in the plain area will, however, experience good conditions.

5 Conclusions

This study dwelt upon a weak point of the performance indices estimating the water supply and resources systems such as Reliability, Resilience, and Vulnerability (RRV) in their threshold values. Traditional methods to quantify RRV used a binary system to recognize deficit periods. This study evaluated a water supply system performance with low, moderate, and high water deficits and stressed that this performance in its real form should affect the RRV and SI estimated values in action, such that these indices are made as close as possible to the real system performance. Two examples with very close performances were examined. In the Example 1, the water demands were fully supplied over the half of the study period, while in the Example 2, the demands were also fully supplied during the half of the study period while suffering a very small deficit of 0.1 unit (0.2%) in allocation of their water requirements. Essentially, both of these performances would, however, practically be the same. Using the traditional method (RRV-Fixed), estimation of the RRV criteria in these two examples led to two different sets of results even though the system's performance was found to remain practically the same. Fuzzy membership functions were employed to alter this weak point. Indeed, the functions were used to estimate the desirability/undesirability with an output value varying from 0 to 1 rather than the traditional binary values. The proposed RRV-Fuzzy method allowed us to use expert knowledge to define a proper desirability function that may be different for each user, demand, or water resource in nature, depending on the importance level imparted by the decision-makers to each one and to the inputs of the functions. Using the traditional performance criteria definitions and formulae, the we introduced this membership function to develop a series of formulae to quantify RRV by the concept of Rel_{Time} and Rel_{Vul} within a more reliable and practical approach named "*Fuzzy Performance Criteria*" and hence the method is named RRV-Fuzzy). Four widely used MFs are investigated and then the Gaussian MF is introduced as the best MF. The performance of the new approach was evaluated using the aforementioned seven theoretical examples. In addition, the method was evaluated based on the current water management (Baseline Scenario) policies in the Zayandehrud basin using a water planning model already available (Safavi et al. 2015). The results obtained by the RRV-Fixed method were then compared with those obtained by the proposed method (RRV-Fuzzy). The results showed the RRV-Fuzzy to outperform the traditional RRV-Fixed method in representing the water supply performance of the system. Fuzzy performance criteria were found to be able to identify the degrees of desirability and thus to evaluate the performance of the system in the proximity of both threshold and intermediate conditions. Accordingly, this method can be used in different studies to better plan and manage the water resources systems addressing the viewpoints of the stakeholders and decision-makers over a variety of case studies.

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Compliance with Ethical Standards

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