ESM 121

Water Science and Management

Exercise 5:

Optimization and Linear Programing



Samuel Sandoval Solis, PhD

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Objective

The objective of this exercise is to provide a set of problems using simple optimization techniques as well as the basics of linear programming using a linear optimization solver in Excel.

Useful Formulas

Find the decision variables, x, that optimizes (maximizes or minimizes) an objective function.

For instance, to minimize:





Steps for Linear Programming

In order to solve linear programing, these are 7 steps that you should follow to identify the maximum or minimum value for the objective function at hand:

- 1. Define the optimization purpose. Is the objective to Maximize or Minimize?
- 2. Define *Objective Function*, or in other words, write the Objective Function into an equation. If the equation is linear (Z=ax₁+bx₂), it can be solved through a linear programming. If not, other techniques of non-linear programming can be used to solve this type of problem.
- Define the Constraints. Write the constraints into inequalities, so they can be used to define the Feasible Region. Notice that the statement "the value of x is greater than (or at least) 20" means x≥20, and the statement "the value of x is smaller than (or less than) 20" means x≤20
- 4. Define the Feasible Region. Use the constraints (inequalities) to bound the feasible region. For constraints with the form "ax₁+bx₂≤C", convert them into equations by dropping the "≤" or "≥" symbol and adding the equal sign "=", such as "ax₁+bx₂=C". Then solve the equation for x₁=0 to obtain the point where the linear equation crosses the x₁ axis. Similarly, solve for x₂=0 to obtain the point where the linear equation cross the x₂ axis.
- 5. Obtain the vertices of the feasible region. Do this by identifying the points in the feasible region and, for the constraints with the form "ax1+bx2≤C", convert them into equations "ax1+bx2=C" and obtain the value of the unknown variable, either x1 or x2, by solving the equation.
- Substitute vertices into the Objective Function. Substitute the values of the variables (x1 and x2) at the vertices into the objective function equation.
- 7. Select the values of the variables that are the maximum or minimum for the objective function, depending on the definition of the objective function (Step 1).

Installing Open Solver - Model

Installing the Open Solver in excel.

- 1.- Copy the OpenSolver21.zip into your computer.
- 2.- Unzip this file

Name	Date modified	Туре	Size
🔲 cbc	2/24/2012 8:45 PM	Application	2,913 KB
CPL License	7/7/2011 12:35 PM	Text Document	12 KB
GNU GPL License	7/7/2011 12:35 PM	Text Document	35 KB
🗿 OpenSolver ChangeLog	9/5/2012 2:45 PM	Microsoft Excel Worksheet	18 KB
📲 OpenSolver	9/6/2012 10:06 PM	Microsoft Excel Add-In	467 KB
📄 ReadMe	11/12/2011 6:55 AM	Text Document	5 KB

3.- Double click on the file: "OpenSolver" which is the add-In

Date modified	Туре	Size
2/24/2012 8:45 PM	Application	2,913 KB
7/7/2011 12:35 PM	Text Document	12 KB
7/7/2011 12:35 PM	Text Document	35 KB
9/5/2012 2:45 PM	Microsoft Excel Worksheet	18 KB
9/6/2012 10:06 PM	Microsoft Excel Add-In	467 KB
11/12/2011 6:55 AM	Text Document	5 KB
	Date modified 2/24/2012 8:45 PM 7/7/2011 12:35 PM 7/7/2011 12:35 PM 9/5/2012 2:45 PM 9/6/2012 10:06 PM 11/12/2011 6:55 AM	Date modifiedType2/24/2012 8:45 PMApplication7/7/2011 12:35 PMText Document7/7/2011 12:35 PMText Document9/5/2012 2:45 PMMicrosoft Excel Worksheet9/6/2012 10:06 PMMicrosoft Excel Add-In11/12/2011 6:55 AMText Document

4.- If a dialogue-window pops up, please select "enable macros"



5.- On the data menu of Excel you should see the Open Solver add-in in the right corner of the menu.

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File Home Insert Page Layout Formulas Da	a Review View Developer Add-Ins DataUp Acrobat	۵ 😮
From Access From Web From Other From Text From Text	5 2↓ 2↓Z V Clear	min z × sy × s z Model Solve v
Get External Data Connections	Sort & Filter Data Tools Outline G Analysis	OpenSolver
▼ (f _x		×

6.- Finally Open the file Ex_5.xlsx

Exercise 1 (Adapted from McKinney).

Based on an annual water allocation of 1,800 acre-feet (AF), an irrigation district wants to know how they can maximize their profits by growing two types of crops, Crop A and Crop B. The profits can be obtained by summing the profit (C_A and C_B) times the acreage (X_A and X_B) for each crop. The irrigation district has constraints. The first constraint is the water use, which is the sum of the water requirement times the acreage for each crop, which must be equal or less than the water allocation (1,800 AF). Second, there are limits to growing one crop or another. The maximum acreage on which to grow crop A is 400 acre, and crop B is 600 acres. Lastly, the acreage must be a positive number for both crops. Table 1 shows the water requirement, expected profit per acre and maximum area cultivated for each crop.

	Crop A	Crop B
Water requirement (Acre feet/acre)	3	2
Profit (\$/acre)	300	500
Max area (acres)	400	600

To be turned in:

- 1) Write down the objective equations and constraints for this problem. Take a look at the "Optimization Presentation", this exercise was explained in this presentation.
- 2) Copy the chart for the feasible region located in Ex 5.xlsx file tab "1 Maximize"
- 3) What was the solution (Maximum value for the objective function Z) in the presentation?

Now, let's work on the Ex_5.xlsx file. Open the tab "1 Maximize". The right side of that tab should look like Figure 1. Every cell that is orange means you must declare some information there.

Variables and Objective Function

First, let's start with the *Variables*. We need to give the model a first guess of the variable, usually any value inside the feasible region and the model will take care to later give us the right result once we have run it. For X_A lets define an initial value of 400 (cell T4=400), and for X_B a value of 600 (cell T5=600), see Figure 2.





Now let's define the Objective function in cell Q5 as the sum of the multiplication of the acreage times the profits for each crop (cell Q5 =T4*W4+T5*W5), see Figure 3.

	TREND) -	- (= 🗙 🖌 f _*	=T4*W4+T	5*W5					
	0	Р	Q	R	S	Т	U	V	W	Х
1	1 Objective Function									
2	Maximize	Z			Variables			Constants		
3	Z	=	C _A *X _A +C _B *X _B		Acreage			Profit		
4	Z	=	300*XA+500*XB		X _A =	400	acres	C _A =	\$300	/acre
5	Z	=	=T4*W4+T5*W5		X _B =	600	acres	C _B =	\$500	/acre
-										



Constraints

Now let's work on the *Constraints*. Constraint 1 specifies that the acreage of crop a (X_A) must be equal to or less than 400 acres. Link cell O10 with the value of XA (cell T4) by typing in cell O10 "=T4"; see Figure 4. Type in cell Q10 the constraint value of 400 (Figure 5).

	TREND	• • (🖹 X 🗸 :	<i>f</i> _x =T4					
	М	N	0	Р	Q	R	S	Т	U
1			Objective	Function					
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	C _A *X _A +C _B *X _B		Acreage		
4	ХВ		Z	=	300*XA+500*XB		X _A =	400	acres
5	840		z	=	\$420,000		X ₈ =	600	acres
6	810								
7	780		Constrain	ts					
8	750		Constrain	t 1					
9	720		XA	5	400				
10	690		=T4	≤					
				-					

Figure 4

	Q10	•	(= × <)	£ 400					
	М	N	0	Р	Q	R	S	т	U
1			Objective	Function					
2	ctive		Maximize	Z			Variables		
3	⊦500*XB		z	=	C _A *X _A +C _B *X _B		Acreage		
4	XB		Z	=	300*XA+500*XB		X _A =	400	acres
5	840		Z	=	\$420,000		X _B =	600	acres
6	810								
7	780		Constraint	s					
8	750		Constraint	1					
9	720		XA	5	400				
10	690		400	≤	400				

Figure 5

Constraint 2 specifies that the acreage of crop A (XA) must be equal to or greater than 0 acres. Link cell O13 with the value of XA (cell T4) by typing in cell O13 "=T4"; see Figure 6. Type in cell Q13 the constraint value of 0 (Figure 7).

_	TREN	D + (<i>f_x</i> =T4					
	М	N	0	Р	Q	R	S	т	U
1			Objective	Function					
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	C _A *X _A +C _B *X _B		Acreage		
4	ХВ		Z	=	300*XA+500*XB		X _A =	400	acres
5	840		Z	=	\$420,000		X _B =	600	acres
6	810								
7	780		Constrain	ts					
8	750		Constrain	t 1					
9	720		XA	≤	400				
10	690		400	≤	400				
11	660		Constraint 2						
12	630		X _A	≥	0				
13	600		=T4	2					
				E	iouro 6				



	QI3	•		er U	-		-	_	
_	M	N	0	Р	Q	R	S	T	U
1			Objective	Function					
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	C _A *X _A +C _B *X _B		Acreage		
4	XB		Z	=	300*XA+500*XB		X _A =	400	acres
5	840		z	=	\$420,000		X _B =	600	acres
6	810								
7	780		Constrain	ts					
8	750		Constrain	1					
9	720		XA	≤	400				
10	690		400	≤	400				
11	660		Constrain	2					
12	630		X _A	≥	0				
13	600		400	2	0				
				F	igure 7				

Constraint 3 specifies that the acreage of crop B (X_B) must be equal to or less than 600 acres. Link cell O16 with the value of X_B (cell T5) by typing in cell O16 "=T5"; see Figure 8. Type in cell Q16 the constraint value of 600 (Figure 9).

	TREND	•	🖲 🗙 🖌 3	<i>f</i> _x =T5					
	М	N	0	Р	Q	R	S	Т	U
1			Objective	Function					
2	ctive		Maximize	Z			Variables		
3	⊦500*XB		Z	=	C _A *X _A +C _B *X _B		Acreage		
4	ХВ		Z	=	300*XA+500*XB		X _A =	400	acres
5	840		Z	=	\$420,000		X _B =	600	acres
6	810								
7	780		Constrain	ts					
8	750		Constrain	t 1					
9	720		X _A	≤	400				
10	690		400	≤	400				
11	660		Constrain	t 2					
12	630		X _A	≥	0				
13	600		400	≥	0				
14	570		Constrain	t 3					
15	540		X _B	≤	600				
16	510		=T5	≤					

Figure 8

	Q16	- (🖲 🗙 🗸 J	£ 600					
	М	N	0	Р	Q	R	S	т	U
1			Objective	Function					
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	C _A *X _A +C _B *X _B		Acreage		
4	XB		Z	=	300*XA+500*XB		X _A =	400	acres
5	840		Z	=	\$420,000		X _B =	600	acres
6	810								
7	780		Constraint	ts					
8	750		Constraint	1					
9	720		X _A	≤	400				
10	690		400	≤	400				
11	660		Constraint	2					
12	630		X _A	2	0				
13	600		400	2	0				
14	570		Constraint	3					
15	540		Х _в	5	600				
16	510		600	≤	600				
]	Figure 9				

Constraint 4 specifies that the acreage of crop B (X_B) must be equal or greater than 0 acres. Link cell O19 with the value of X_B (cell T5) by typing in cell O19 "=T5"; see Figure 10. Type in cell Q19 the constraint value of 0 (Figure 11).

	TREND \checkmark $(\land \checkmark f_x) = T5$								
	М	N	0	Р	Q	R	S	Т	U
1			Objective	Function					
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	C _A *X _A +C _B *X _B		Acreage		
4	ХВ		Z	=	300*XA+500*XB		X _A =	400	acres
5	840		Z	=	\$420,000		X _B =	600	acres
6	810								
7	780		Constraint	s					
8	750		Constraint	1					
9	720		X _A	≤	400				
10	690		400	≤	400				
11	660		Constraint	2					
12	630		X _A	≥	0				
13	600		400	2	0				
14	570		Constraint	3					
15	540		X _B	≤	600				
16	510		600	≤	600				
17	480		Constraint	4					
18	450		X _B	≥	0				
19	420		=T5	≥					
					auna 10				



Constraint 5 specifies that the sum of the water use in crop A (3^*X_A) and crop B (2^*X_B) must be equal or less than 1800 acre-feet. In cell O22 we have to write this equation as follow: "=Z4*T4+Z5*T5"; see Figure 12. Type in cell Q19 the constraint value of 1800 (Figure 13).

SUN	1 ×	: × 🗸	<i>f_x</i> =Z4*T4+	Z5*T5											
	М	N	0	Ρ	Q	R	S	Т	U	V	W	Х	Y	Z	AA
1			Objective	unction											
2	ctive		Maximize	Z											
3	F200*XB		Z	=	$C_A * X_A + C_B * X_B$		Variables			Profit			Crop duty		
4	ХВ		Z	=	300*XA+500*XB		X _A =	400	acres	C _A =	\$300	/acre	Crop A=	3.0	AF/acre
5	840		z	=	\$420,000		X _B =	600	acres	C _B =	\$500	/acre	Crop B=	2.0	AF/acre
6	810														-
7	780		Constraints	5			1200					6			
8	750		Constraint 2	1							nstraint 1	-Cons	straint 2		
9	720		X _A	≤	400		x.	>0			nstraint 3		straint 4		
10	690		400	≤	400		1000	→	X.<400		Instraint 5		cuve		
11	660		Constraint 2	2				رد _ر	←	- 30	lution				
12	630		X _A	≥	0		800	TT X							
13	600		400	≥	0		@								
14	570		Constraint	3			ě 4	100							
15	540		X _B	≤	600		B 600	<-<600			X-<600				
16	510		600	≤	600		¥	-B=000			AB-000	,			
17	480		Constraint 4	4			× 100								
18	450		X _B	≥	0		400	Feasible							
19	420		600	≥	0			Region		J.C.					
20	390		Constraint !	5			200			T x y					
21	360		3X _A +2X _B	≤	1800				X	TTP.					
22	330	:	=Z4*T4+Z5*T	5			X,	^{,≥0}	X _A ≤400	K 600	X _R ≥0				
23	300						0	20	0 400	600	800	1000	1200		
24	270							20	. 100	X₄ (acre	es)	1000	1200		
25	240														
26															
							Figur	e 12							

	Q22	- (🖯 🗙 🗸 J	£ 1800					
	М	N	0	Р	Q	R	S	Т	U
1			Objective	Function					
2	ctive		Maximize	Z			Variables		
3	⊦500*XB		Z	=	C _A *X _A +C _B *X _B		Acreage		
4	ХВ		Z	=	300*XA+500*XB		X _A =	400	acres
5	840		Z	=	\$420,000		X _B =	600	acres
6	810								
7	780		Constraint	s					
8	750		Constraint	1					
9	720		X _A	≤	400				
10	690		400	≤	400				
11	660		Constraint	2					
12	630		X _A	≥	0				
13	600		400	≥	0				
14	570		Constraint	3					
15	540		Х _в	≤	600				
16	510		600	≤	600				
17	480		Constraint	4					
18	450		Х _в	≥	0				
19	420		600	≥	0				
20	390		Constraint	5					
21	360		3X _A +2X _B	≤	1800				
22	330		2400	٤	1800				

Figure 13

Save your spreadsheet (Ctrl+S).

Defining the Optimization Linear Program

Now, let's define the linear program model. Go to the Data menu and click on "Min Z $x \le y x=2$ Model" icon (Figure 14)

Formulas Data Review View Developer Add-Ins Datup Acrobat Image: Connections of Properties esh 21 Image: Connection of Properties esh Image: Connect				Ex_7_V2 - Microsoft Excel	_			
Image: Connections of Properties esh in Software Image: Connections of Clear of Cle	Formulas Data	Review	View Developer	Add-Ins DataUp Acrob	pat			∧ 🕜 🗖 🗗 🗙
Columns Duplicates Validation * Analysis * * * OpenSolver *	Connections Properties esh	A Z↓ AZ Z↓ Sort	Filter	Text to Remove Data Columns Duplicates Validation *	Consolidate What-If Analysis •	Group Ungroup Subtotal	Data Analysis	min z × s y × s y Nodel Show/Hide Model Quick Solve Quick Solve OpenSolver *
Connections Sort & Filter Data Tools Outline File Analysis OpenSolver	Connections	So	rt & Filter	Data Tools		Outline	🗟 Analysis	OpenSolver

Figure 14

A dialogue window will appear, displaying the "Open Solver - Model" (Figure 15).

What is AutoModel?	AutoModel
AutoModel is a feature of OpenSolver the structure of the spreadsheet. It will openSolver or Solver. Note that you do lease note that AutoModel will replace	tries to automatically determine the problem you are trying to optimise by the observing it is best guess into a Solver model, which you can then edit in this window and solve w nave to use this feature: the model can still be built manually. model in this window, but won't save it to the sheet until you click Save Model.
Dbjective cell:	maximise
/ariable cells:	
Constraints:	
<add constraint="" new=""></add>	
	Add constraint Cancel
	Delete selected constraint
	✓ Make unconstrained variable cells non-negative
Nodify a constraint: select, make chang Add a constraint: select "Add new cons	then click "Update constraint". it", enter the new constraint's details, then click "Add Constraint"

Figure 15

Go to the Objective cell section, and click on the icon to browse the objective cell (Figure 15). Select the cell Q5, which is the cell with the equation of the objective function (Figure 16).

Н		J	К	L	M	N	0	Р	Q
		m=	-1.5				Objectiv	e Func	tion
9				-	2	Y)	Maximiz	Z	
Op	enSolv	/er - Mo	del		(B		z	=	CA"XA+CB"X
11	Maximiz	e'!\$O\$5					Z	=	300°XA+500°X
			_		_		z	-	\$360,000
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100	0	100	750	100	660		Constrai	ints	
150	0	150	675	150	630		Constrain	t1	
	-				 	1.0	· · ·		· · · · ·

Figure 16

When you come back to the dialogue window, make sure to select "maximize" (Figure 17).

OpenSolver - Model	
What is AutoModel?	AutoModel
AutoModel is a feature of OpenSolver that tries to automatically determine the structure of the spreadsheet. It will turn its best guess into a Solver m OpenSolver or Solver. Note that you don't have to use this feature: the m Please note that AutoModel will replace the model in this window, but won	the problem you are trying to optimise by the observing odel, which you can then edit in this window and solve with odel can still be built manually. 't save it to the sheet until you dick Save Model.
Objective cell: 1 Maximize'!\$Q\$5	C minimise C target value: 0
Variable cells:	_

Figure 17

Now click on the icon to choose the cells that will be the variables (Figure 18). Select cells T4 and T5 (Figure 19).

OpenSolver - Mo	del	Statement (State	and in such that		-	10		×
What is AutoM	odel?				A	utoModel		
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Objective cell:	1 Maximize'!\$Q\$5	_ •	° maximise	() minimise	⊖ targ	et value:	0	
Variable cells:								
Figure 18								
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n= -1.5		Objecti	ve Functio	n				
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750	000 000	C .						
		F	igure 19	9				

Now, we will add the constraints to the model. Let's start with the first constraint ($X_A \le 400$), go to the constraints section and click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O10 (Figure 21), then go back to the dialogue menu. Select the symbol of equal or less than " \le " (Figure 22). Now click on the icon to select the right side of the inequality (Figure 23), cell Q10 (Figure 24) and go back the dialogue window. Click on "Add Constraint" (Figure 25). The inequality Cell O10 \le Cell Q10 should have appeared in the left side of the Constraints section of the dialogue window (Figure 26). We just have finished declaring the first constraint.

OpenSolver - Mo	del	Column Name	-	Summer States	×
What is AutoM	odel?			AutoModel	
AutoModel is a fe the structure of t OpenSolver or So Please note that <i>i</i>	ature of OpenSolver that tries to a he spreadsheet. It will turn its bes lver. Note that you don't have to a AutoModel will replace the model in	automatically determin t guess into a Solver n use this feature: the n n this window, but won	e the problem yo odel, which you odel can still be 't save it to the	ou are trying to optim u can then edit in this built manually. sheet until you click S	ise by the observing window and solve with ave Model.
Objective cell:	'1 Maximize'!\$Q\$5	- • maximise	C minimise	○ target value:	0
Variable cells:	'1 Maximize'!\$T\$4:\$T\$5				_
Constraints:					
<add cons<="" new="" td=""><td>straint></td><td></td><td></td><td></td><td>_ = _</td></add>	straint>				_ = _
			Ad	d constraint	Cancel
				Delete selected c	onstraint
			✓ Make	unconstrained variab	le cells non-negative

Figure 20

Н	1	J	K	L	M	N	0	P	Q	B
_		m=	-1.5		_		Objectiv	e Func	tion	
On	enSol	ver - Mo	del		2	X	Maximiz	Z		
	Chool	rei me	uci	-	_		Z	=	CA"XA+CB"	Хв
1	Maximi	ze'!\$0\$1	0				Z	=	300"XA+500"	ĸв
6	U	U	900	U	720		z	=	\$360,000	
50	0	50	825	50	690					
100	0	100	750	100	660		Constra	ints		
150	0	150	675	150	630		Constrain	e 1		
200	0	200	600	200	600		X _A	≤	400	
250	0	250	525	250	570		200	≤	400	
200	~	000	450	000	E 40		- · ·	~		

Figure 21

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Figure 22

OpenSolver - Model	
What is AutoModel?	AutoModel
AutoModel is a feature of OpenSolver that tries to automatically determin the structure of the spreadsheet. It will turn its best guess into a Solver of OpenSolver or Solver. Note that you don't have to use this feature: the r Please note that AutoModel will replace the model in this window, but wor	e the problem you are trying to optimise by the observing model, which you can then edit in this window and solve with model can still be built manually. n't save it to the sheet until you click Save Model,
Objective cell: 11 Maximize'!\$Q\$5	C minimise C target value:
Variable cells: '1 Maximize'!\$T\$4:\$T\$5	
Constraints:	
<add constraint="" new=""></add>	'1 Maximize'!\$O\$10 _ = = •
	Add constraint Cancel
	Delete selected constraint



Н	1	J	K	L	M	N	0	Р	Q
		m=	-1.5				Objectiv	e Func	tion
Const	Constraint 4 Constraint 5		Objective			Maximiz	Z		
XE	≥0	3XA+2>	<b≤1800< td=""><td>300°XA</td><td>+500°XB</td><td></td><td>Z</td><td>=</td><td>Ca"Xa+CB"X</td></b≤1800<>	300°XA	+500°XB		Z	=	Ca"Xa+CB"X
XA	XB	XA	ХB	XA	XB		Z	=	300"XA+500"XI
0	0	0	900	0	720		Z	=	\$360,000
50	0	50	825	50	690				
10-			-	1000	9	v D	Constra	ints	
1 0	penSo	ver - Mo	odel		L I		Constrain	t1	
2 🗔	Maxim	ize'i ¢O¢1	ol				X _A	≤	400
a L	mdXIII	ize : şQşı					200	≤	400
300	0	300	450	300	540		Constrain	+2	

Figure 24

OpenSolver - Mo	del	7		1.25	AutoModel	
AutoModel is a fe the structure of t OpenSolver or So Please note that	ature of OpenSolver that t he spreadsheet. It will turn lver. Note that you don't h AutoModel will replace the i	ries to automa its best guess ave to use this model in this w	tically determin i into a Solver r i feature: the r indow, but wor	e the problem you nodel, which you nodel can still be I't save it to the	ou are trying to optimi can then edit in this built manually, sheet until you click S	ise by the observing window and solve wit ave Model,
Objective cell:	'1 Maximize'!\$Q\$5	_	(maximise	() minimise	○ target value:	0
Variable cells:	'1 Maximize'!\$T\$4:\$T\$5					_
Constraints:						
<add cons<="" new="" td=""><td>straint></td><td></td><td></td><td>1 Maxi</td><td>mize'!\$Q\$10 mize'!\$Q\$10 d constraint</td><td>_ = V</td></add>	straint>			1 Maxi	mize'!\$Q\$10 mize'!\$Q\$10 d constraint	_ = V

Figure 25

OpenSolver - Model					
What is AutoModel? AutoModel is a feature of OpenSolver that tries to automatically determine the structure of the spreadsheet. It will turn its best guess into a Solver mo OpenSolver or Solver. Note that you don't have to use this feature: the mo Please note that AutoModel will replace the model in this window, but won't	AutoModel the problem you are trying to optimise by the observing del, which you can then edit in this window and solve with del can still be built manually. save it to the sheet until you dick Save Model.				
Objective cell: 1)Maximize'!\$Q\$5 _ (~ maximise	⊖ minimise ⊖ target value: 0				
Variable cells: 1)Maximize'!\$T\$4:\$T\$5	_				
Constraints:					
<add constraint="" new=""> \$O\$10 <= \$Q\$10</add>	_ <= ▼ Add constraint Cancel				
Figure 26)				

To declare the second constraint ($X_A \ge 0$), click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O13 (Figure 27), then go back to the dialogue menu. Select the symbol of equal or greater than " \ge " (Figure 22). Now click on the icon to select right-side part of the inequality (Figure 23), select cell Q13 (Figure 29) and go back the dialogue window. Click on "Add Constraint". The inequality Cell O13 \ge Cell Q13 should have appeared in the left side of the Constraints section of the dialogue window (Figure 30).

-		J	К	L	М	N	0	P	Q			
		m=	-1.5				Objecti	ve Func	tion			
onst	onstraint 4 Constraint 5		raint 5	Objective			Maximiz	Z				
XE	≥0	3XA+2X	(B≤1800	300°XA	+500°XB		Z	=	Ca"Xa+CB"XB			
A:	XB	XA	XB	XA	XB		Z	=	300"XA+500"XB			
)	0	0	900	0	720		z	=	\$360,000			
i0	0	50	825	50	690							
00	0	100	750	100	660		Constra	ints				
50	0	150	675	150	630		Constrain	nt 1				
00	0	200	600	200	600		×a	≤	400			
50	0	250	525	250	570		200	≤	400			
70-	- 0-	- 200	450	200	F40	~ D	Constrain	nt 2				
0	penSo	lver - M	odel		8	<u> </u>	\times_{A}	≥	0			
	1 Maximize'!\$0\$13 Constraint 3											
1	~	PAA	450						000			





-		J	К	L	M	N	0	P	Q		
		m=	-1.5				Objecti	ve Func	tion		
onst	onstraint 4 Constraint 5		Obje	Objective		Maximiz	Z				
XE	≥0	3XA+2>	(B≤1800	300°XA	+500°XB		Z	=	CA"XA+CB"XB		
(A	XB	XA	XB	XA	XB		Z	=	300"XA+500"XB		
0	0	0	900	0	720		z	=	\$360,000		
50	0	50	825	50	690						
D0	0	100	750	100	660		Constra	ints			
50	0	150	675	150	630		Constrain	nt 1			
00	0	200	600	200	600		Xe	≤	400		
50	0	250	525	250	570		200	≤	400		
ſς	penSc										
_ 9	X _A ≥										
- d '	1 Maxin	nize'!\$Q\$	13		.	200	2	0			
	Constraint 3										



OpenSolver - Mo	del	P 10-1-1-		1 2 1	distant.	— X
What is AutoMe	odel?				AutoModel	
AutoModel is a fea the structure of the OpenSolver or Sol Please note that A	ature of OpenSolver that tries ne spreadsheet. It will turn its lver. Note that you don't have AutoModel will replace the mo	s to automatical best guess int to use this fea del in this windo	lly determin o a Solver n ature: the n ow, but wor	e the problem nodel, which y nodel can still 't save it to t	you are trying to optim rou can then edit in this be built manually. he sheet until you click S	ise by the observing window and solve with Save Model.
Objective cell:	'1 Maximize'!\$Q\$5	- •	maximise	() minimise	C target value:	0
Variable cells:	'1 Maximize'!\$T\$4:\$T\$5					_
Constraints:						
<add cons<br="" new="">\$0\$10 = \$Q\$1 \$0\$13 >= \$Q\$</add>	straint> 0 :13					_ >= •
					Add constraint	Cancel
					Delete selected o	constraint

Figure 30

To declare the third constraint ($X_B \le 600$), click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O16 (Figure 31), then go back to the dialogue menu. Select the symbol of equal or greater than " \le " (Figure 32). Now click on the icon to select right-side part of the inequality (Figure 23), select cell Q16 (Figure 33) and go back the dialogue window. Click on "Add Constraint". The inequality of cell O16 \le Cell Q16 should have appeared in the left side of the Constraints section of the dialogue window (Figure 34).

Н	1	J	K	L	M	N	0	P	Q	
		m=	-1.5				Objective Func		tion	
Const	raint 4	Const	raint 5	Objective			Maximiz	Z		
XE	XB≥0 3XA+2X8		(B≤1800	300°XA	+500°XB		Z	=	CA"XA+CB"	К _в
XA	XB	XA	XB	XA	XB		Z	=	300"XA+500">	KΒ
0	0	0	900	0	720		Z	=	\$360,000	
50	0	50	825	50	690					
100	0	100	750	100	660		Constra	ints		
150	0	150	675	150	630		Constrain	e 1		
200	0	200	600	200	600		X _A	≤	400	
250	0	250	525	250	570		200	≤	400	
300	0	300	450	300	540		Constrain	t 2		
350	0	350	375	350	510		X _A	≥	0	
100	0	400	300	400	480	_	200	2	0	
0	henSol	ver - Mo	del		2	X	Constrain	t 3		
	Jeni Jon	ver ivie	uci	1000	_		X _B	≤	600	
1	Maximi	ze'!\$O\$1	6			600	≤	600		
		-		-		Constrain	t 4			

Figure 31





-		J	K	L	M	N	0	P	Q
		m=	-1.5				Objectiv	e Fund	stion
onst	nstraint 4 Constraint 5		raint 5	Objective			Maximiz	Z	
XE	≥0	3XA+2>	K B≤1 800	300°XA	+500°XB		z	=	CA"XA+CB"X
:A	XB	XA	XВ	XA	XB		Z	=	300"XA+500"X
)	0	0	900	0	720		z	=	\$360,000
i0	0	50	825	50	690				
)0	0	100	750	100	660		Constrai	ints	
50	0	150	675	150	630		Constrain	t1	
D0	0	200	600	200	600		X _A	≤	400
50	0	250	525	250	570		200	≤	400
D0	0	300	450	300	540		Constrain	t2	
50	0	350	375	350	510		X _A	≥	0
00	0	400	300	400	480		200	2	0
Ío	penSc	olver - M	odel	Constrain	t3				
1.2		-		1000		X _B	≤	600	
	1 Maxin	nize'!\$Q\$	16		600 ≤ 600				
1	_	_	-				Constrain	t 4	



OpenSolver - Model	
What is AutoModel?	AutoModel
AutoModel is a feature of OpenSolver that tries to automatically determine the structure of the spreadsheet. It will turn its best guess into a Solver mo OpenSolver or Solver. Note that you don't have to use this feature: the mo Please note that AutoModel will replace the model in this window, but won't	the problem you are trying to optimise by the observing del, which you can then edit in this window and solve with del can still be built manually. save it to the sheet until you click Save Model.
Objective cell: 1 Maximize'! \$Q\$5	C minimise C target value:
Variable cells: 1'1 Maximize'!\$T\$4:\$T\$5	_
Constraints:	
<add constraint="" new=""> \$0\$10 = \$Q\$10 \$0\$13 = \$Q\$13 \$<mark>\$0\$16 <= \$Q\$16</mark></add>	Add constraint Cancel

Figure 34

19

To declare the fourth constraint ($X_B \ge 0$), click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O19 (Figure 35), then go back to the dialogue menu. Select the symbol of equal or greater than " \ge " (Figure 36). Now click on the icon to select right-side part of the inequality (Figure 23), select cell Q19 (Figure 37) and go back the dialogue window. Click on "Add Constraint". The inequality Cell O19 \ge Cell Q19 should have appeared in the left side of the Constraints section of the dialogue window (Figure 38).



Figure 35





Н	1	J	K	L	M	N	0	Р	Q
		m=	-1.5				Objective Fun		tion
onst	raint 4	Const	raint 5	Objective			Maximiz	Z	
XE	≥0	3XA+2XB≤1800		300"XA+500"XB			z	=	CA'XA+CB'X
<a td="" <=""><td>XB</td><td>XA</td><td>XВ</td><td>XA</td><td>XB</td><td></td><td>Z</td><td>=</td><td>300"XA+500"XE</td>	XB	XA	XВ	XA	XB		Z	=	300"XA+500"XE
0	0	0	900	0	720		Z	=	\$360,000
50	0	50	825	50	690				
00	0	100	750	100	660		Constrai	nts	
50	0	150	675	150	630		Constraint	1	
00	0	200	600	200	600		X _A	≤	400
50	0	250	525	250	570		200	≤	400
00	0	300	450	300	540		Constraint	2	
50	0	350	375	350	510		X _A	≥	0
00	0	400	300	400	480		200	≥	0
50	0	450	225	450	450		Constraint	3	
00	0	500	150	500	420		Xe	≤	600
7					100	× D	600	≤	600
0	penSo	lver - M	odel		B		Constraint	4	
1	Maria						Xe	≥	0
1Ľ	L Maxim	iize ! şQş	19				600	≥	0
डेगा		750	-775	7511	270		Constraint	5	

Figure 37

OpenSolver - Mo	del	-	-	-	-	×
What is AutoM	odel?				AutoModel	
AutoModel is a fe the structure of the OpenSolver or So Please note that A	ature of OpenSolver that trie he spreadsheet. It will turn it lver. Note that you don't hav AutoModel will replace the mo	s to auton s best gue re to use t idel in this	natically determin iss into a Solver n his feature: the n window, but won	e the problem y nodel, which you nodel can still be 't save it to the	ou are trying to optim a can then edit in this built manually. sheet until you click S	ise by the observing window and solve with ave Model.
Objective cell:	'1 Maximize'!\$Q\$5	-	(maximise	() minimise	○ target value:	0
Variable cells:	'1 Maximize'!\$T\$4:\$T\$5					_
Constraints:						
<add cons<br="" new="">\$0\$10 = \$Q\$1</add>	straint> 0					_ >= •
\$0\$13 >= \$Q\$ \$0\$16 <= \$Q\$	13 16			Í		
\$U\$19 >= \$Q\$	51 <u>9</u>			Ad	ld constraint	Cancel
					Delete selected c	onstraint
				🔽 Make	unconstrained variab	le cells non-negative

Figure 38

To declare the fifth constraint $(3X_4+2X_B \le 1800)$, click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O22 (Figure 39), then go back to the dialogue menu. Select the symbol of equal or greater than " \le " (Figure 40). Now click on the icon to select right-side part of the inequality (Figure 23), select cell Q22 (Figure 41) and go back the dialogue window. Click on "Add Constraint". The inequality of cell O22 \ge Cell Q22 should have appeared in the left side of the Constraints section of the dialogue window (Figure 42).

Н	- 1	J	K	L	M	N	0	Р	Q
		m=	-1.5				Objectiv	e Func	tion
Const	raint 4	Const	raint 5	Objective			Maximiz	Z	
XE	≥0	3XA+2>	KB≤1800	300"XA+500"XB			z	=	Ca"Xa+CB"X
XA	ХB	XA	XB	XA	XB		Z	=	300"XA+500"XI
0	0	0	900	0	720		z	=	\$360,000
50	0	50	825	50	690				
100	0	100	750	100	660		Constrai	ints	
150	0	150	675	150	630		Constrain	t 1	
200	0	200	600	200	600		X _A	≤	400
250	0	250	525	250	570		200	≤	400
300	0	300	450	300	540		Constrain	12	
350	0	350	375	350	510		X _A	≥	0
400	0	400	300	400	480		200	2	0
450	0	450	225	450	450		Constrain	:3	
500	0	500	150	500	420		X _B	≤	600
550	0	550	75	550	390		600	≤	600
600	0	600	0	600	360		Constrain	t 4	
650	0	650	-75	650	330		X _B	≥	0
700_	0	700	-150	700	300	-	600	≥	0
(On	enSol	ver - Mo	del		2	X	Constrain	:5	
_		-		-		$3X_{A}+2X_{B}$	≤	1800	
1	Maximiz	ze'!\$O\$2	2			1800	≤	1800	
S			-						

Figure 39



Figure 40

+		JK		L M N		N	0	Р	Q		
m=		-1.5				Objectiv	/e Func	Function			
onst	raint 4	Constraint 5		Objective			Maximiz	Z			
XE	≥0	3XA+2XB≤1800		300"XA+500"XB			Z	=	CA"XA+CB"		
:A	XВ	XA	XB	XA XB			Z	=	300"XA+500"X		
С	0	0	900	0 720			Z	=	\$360,000		
i0	0	50	825	50	690						
00	0 100 750		750	100 660			Constra	ints			
50	0	150 675		150 630			Constrain	e 1			
00	0	200	600	200	600		X _A	≤	400		
50	0	250	525	250	570		200	≤	400		
D0	0	300	300 450		300 540		Constrain	t2			
50	0	350	375	350	510		X _A	2	0		
D0	0	400	300	400	480		200	2	0		
50	0	450	225	450 450			Constraint 3				
00	0	500	150	500	420		XB	≤	600		
50	0	550	75	550	390		600 ≤		600		
D0	0	600	0	600 360			Constraint 4				
50	0	650	-75	650	330		XB	2	0		
	~			600	≥	0					
0	penso	Iver - M	odel	Constrain							
1	Maxim	ize'l¢∩¢	22		$3X_{A}+2X_{B}$	≤	1800				
Ľ	- HUAIN	nze : şQşi	- 4				1800	S	1800		

Figure 41

OpenSolver - Mo	del		states in such	-	and the second s	×				
What is AutoM	odel?			AutoModel						
AutoModel is a fe the structure of t OpenSolver or So Please note that i	ature of OpenSolver ti he spreadsheet. It will lver. Note that you do AutoModel will replace	nat tries to autor turn its best gue n't have to use t the model in this	natically determin ess into a Solver r his feature: the r window, but wor	e the problem yo nodel, which you nodel can still be 't save it to the	ou are trying to optim can then edit in this built manually. sheet until you click S	ise by the observing window and solve with ave Model.				
Objective cell:	\$Q\$5	_	(maximise	C minimise	C target value:	0				
Variable cells:	\$T\$4:\$T\$5					_				
Constraints:										
<add cons<br="" new="">\$0\$10 <= \$Q\$ \$0\$13 >= \$Q\$ \$0\$16 <= \$Q\$</add>	straint> \$10 \$13 \$16			\$0\$22		_ <= ▼				
\$0\$19 >= \$0\$ \$0\$22 <= \$0\$	19 22			Upda	ate constraint	Cancel				
					Delete selected o	onstraint				
				▼ Make	unconstrained variab	le cells non-negative				

Figure 42

Check the box that says "List constraints and shadow prices in a table with top left cell:" and select the cell "AC8" (Figure 43)

OpenSolver - Mo	odel			×							
What is AutoMo AutoModel is a fea the structure of th OpenSolver or Sol	odel? ature of OpenSolver that tries t e spreadsheet. It will turn its b ver. Note that you don't have t	AutoModel the problem you are trying to optimise by the observing odel, which you can then edit in this window and solve with odel can still be built manually.									
Please note that A Objective cell:	AutoModel will replace the mode	save it to the sheet until you dick Save Model.									
Variable cells: \$T\$4:\$T\$5											
Constraints:											
<pre><add \$o\$10="" \$o\$13="" <="\$Q\$" cons="" new="">= \$Q\$ \$O\$16 <= \$Q\$ \$O\$16 <= \$Q\$ \$O\$19 >= \$Q\$ \$O\$22 <= \$Q\$</add></pre>	traint> 10 13 16 19 22		\$0\$22 \$Q\$22 Update constraint	■ <= ▼ ■ Cancel							
			Delete selected constraint								
			Make unconstrained va	ariable cells non-negative							
Modify a constraint: select, make changes, then click "Update constraint". Add a constraint: select "Add new constraint", enter the new constraint's details, then click "Add Constraint"											
Shadow Prices:	Shadow Prices: V List constraints and shadow prices in a table with top-left cell:										
✓ Show model at	fter saving	Options	Save Model Cancel								

Figure 43

Click on "Save Model" (Figure 44). A Series of colorful rectangles should have appeared in your Spreadsheet (Figure 45). Notice that the Objective Function (cell Q5) has a "Max" label on top of it. Also, notice that the variables have a pink rectangle on top. In addition, for each constraint the right and left side of the equations/inequality are related with a line and a symbol (" \geq ", " \leq " or "="), Double check that these symbols are correct.

penSolver - Mo	del	-		1	12.24	l in	— X
What is AutoMe	odel?				Aut	oModel	
AutoModel is a fei the structure of the OpenSolver or So Please note that A	ature of OpenSolver that tries he spreadsheet. It will turn its lver. Note that you don't hav AutoModel will replace the mo	s to automatically best guess into a e to use this featu del in this window	determine f a Solver mo ure: the mo , but won't	the problem del, which y del can still save it to th	you are trying ou can then edi be built manuali ie sheet until yo	to optimi it in this y. ou click S	se by the observing window and solve with ave Model.
Objective cell:	'1 Maximize'!\$Q\$5	_ (* mi	aximise	⊖ minimise	C target	value:	0
Variable cells:	'1 Maximize'I\$T\$4:\$T\$5						_
Constraints:							
<add cons<br="" new="">\$0\$10 = \$Q\$1 \$0\$13 >= \$0\$</add>	straint> 0 \$13						_ >= •
\$O\$16 <= \$Q \$O\$19 >= \$Q	16 19						_
\$0\$22 >= \$Q\$	22				Add constraint		Cancel
					Delete se	lected o	onstraint
				🔽 Mak	e unconstraine	d variabl	e cells non-negative
Modify a constrair Add a constraint:	nt: select, make changes, the select "Add new constraint",	n click "Update co enter the new cor	nstraint". nstraint's de	etails, then	lick "Add Const	raint"	
Shadow Prices:	List constraints and shad	ow prices in a tab	le with top-	left cell:			-
Show model a	fter saving	Option	15	s	ave Model		Cancel

Figure 44



Running the Linear Programming Model

It is time to run the model!!! Click on the Solve icon (Figure 46) of the Open Solver. After the screen blinks you will notice that numbers have changed. The maximum profit that can be obtained are **\$360,000** (Cell Q5)!!! The values for the variable have changed, X_A =200 and X_B =600, as we calculated in the presentation at class!!!



To be turned in:

- 1) Take a look at the shadow values. For the constraint in cells $O16 \le Q16$ (X_B \le 600) the shadow value is equal to -300, what does it mean? *Do the following to get a hint for the answer of this question*: Change the value of cell Q16 to 601 and run the model. What is the new value of the objective function (cell Q5)? Now change it to 602, and take a look at the objective function value. How much the objective value changed for 601? And for 602? And for 603? Does the shadow price tell you how much the objective function will change by a unit increase in the constraint?
- 2) A screenshot of the model, like Figure 45.
- 3) Now let's invert the values so the Profits for C_A (cell W4) are equal to \$500/acre and for C_B (cell W5) are equal to \$300/acre. Click on Solve. What are the results for X_A and X_B? Does the value of the profits change the results for the optimal solution? If so, do you think changing the market prices can change the optimal solution of linear systems?
- 4) A screenshot of the new model with the profit values changed.

Exercise 2 (Adapted from Loucks and van Beek)

A city planner wants to know the minimum cost of waste removal (WR) at Sites 1 and 2 upstream of a recreational park (Figure 47). The cost of waste removal can be obtained by summing the cost of 100% (C_1 and C_2) times the fraction of removal for each site (X_1 and X_2). According to a pre-design, the fraction of WR at site 1 (X_1) must be equal to or greater than 0.8 but equal to or less than 1. The fraction of WR at site 2 (X_2) must be equal to or less than 1. There is another constraint expressed as the equation: $X_1 + 1.3 X_2 \ge 1.8$.



Use the "Optimization" presentation in class to solve this problem. The optimization model is as follows:

```
Minimize Z

Z = C_1(x_1) + C_2(x_2)
Where: C_1 = 200; C_2 = 100

Subject to

x_1 \ge 0.8

x_1 \le 1

x_2 \ge 0

x_2 \le 1

x_1 + 1.3x_2 \ge 1.8
```

Using the Excel spreadsheet Ex_5.xlsx in Tab "2) Minimize", create a linear program using the Open Solver Model. Don't forget to select "<u>Minimize</u>" instead of "Maximize" (Figure 17) when creating the linear model.

To be turned in:

- 1) A screenshot of your linear model, such as Figure 44.
- What is the minimum cost of removal (the optimum value of the objective function) obtained once the model has been run? How do your results compare with the ones in the presentation (Slide 20, minimum cost of removal equal to \$238K)

Exercise 3 The perfect outfit!

Summer is coming and you are looking for the perfect outfit to wear, while, at the trying not to over spend. You have a total budget of \$130. This is a suggested number of minimum and maximum number of pieces that you want to buy for each article.

Article	Minimun # of pieces	Maximum # of Pieces				
Jeans (C _{Jeans})	1	3				
T-Shirts ($C_{T-shirts}$)	1	4				

Other constraints are that you want to buy equal or more T-shirts than jeans ($C_{T-shirts} \ge C_{Jeans}$), and that the total amount spent should not be more than your dedicated budget \$130.

To be turned in:

Following the seven steps of linear programing (page 4 of this exercise)

- 1) Write down the *objective function* and the constraints
- 2) Draw the *feasible region*. You can use excel, or a simple piece of paper and scan it, or some engineering paper.
- 3) Obtain the vertices of the feasible region. Substitute the values of the vertices into the objective function. Submit a table showing the vertices of the feasible region as well as the value of the objective function for each pair of C_{Jeans} and C_{T-shirts} values (see slides 13 and 19 of the presentation of "Optimization").
- 4) Based on the previous analysis, what is the combination Jeans and T-shirts that you should buy?

Now, let's use the Open Solver model of excel to solve this problem. Use the file Ex_5.xlsx - tab "3 Maximize" to create a linear optimization model. You can use as initial values for C_{Jeans} (cell G5) and $C_{T-shirt}$ (cell G6) 1 and 1, respectively. Calculate amount spend in cell D6 (for the objective function) and B23 (for the constraint) by multiplying the cost of the jeans times the number of jeans (J5xG5) plus the cost of T=shirts times the number of T-shirts (J6xG6). When creating the linear optimization model in Open Solver, don't forget to select "Maximize" (Figure 17). In addition, you will have to declare in the optimization model that the number of jeans C_{Jeans} (cell G5) and T-shirts $C_{T-shirts}$ (cell G6) should be integers (see figure 48).

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Figure 48

- 5) Turn in a screenshot of your linear model, similar to Figure 44.
- 6) How much will you spend (the optimum value of the objective function) obtained once the model has been run (cell D5)?
- 7) How many Jeans and T-shirts you can buy C_{Jeans} (cell G5) and $C_{T-shirts}$ (cell G6) for the maximum amount of money spent given the constraints?
- 8) How do these results compare with the same analysis done before in question 4 of this problem (see above)? Are these results similar?
- 9) What are the shadow prices for the different constraints? What is the maximum shadow price? If you change any constraint "1 unit", how much will the Objective function will change? Are there any biding constraints?

Exercise 4 (Adapted from Loucks and van Beek)

Consider a water-using industry that plans to obtain water from a groundwater aquifer. Two wellfield sites have been identified, A and B. The objective of this industry is to minimize the cost of pumping from the wells at sites A and B. The cost of pumping can be obtained by summing the cost of pumping at site A plus site B (*Cost* A + Cost B); both of them depend on the water extracted Q_A and Q_B from sites A and B, respectively. Cost A is expressed by the equation:

$$Cost A = 8 + \frac{40 - 8}{17} * Q_A$$

Cost B is expressed by the equation:

$$Cost B = 15 + \frac{26 - 15}{13} * Q_B$$

For the well at site A, the water extracted (Q_A) must be equal to or greater than 0. The maximum sustainable groundwater extraction at site A is equal to or less than 17 acre-feet/year.

For the well at site *B*, the water extracted (Q_B) must be equal to or greater than 0. The maximum sustainable groundwater extraction at site *B* is equal to or less than 13 acre-feet /year. The water required to satisfy the industry is 22 acre-feet/year.



Figure 49

To be turned in:

Following the seven steps of linear programing (page 4 of this exercise)

- 1) Write down the *objective function* and the constraints
- 2) Draw the *feasible region*. You can use excel, or a simple piece of paper and scan it, or some engineering paper.
- 3) Obtain the vertices of the feasible region. Substitute the values of the vertices into the objective function. Submit a table showing the vertices of the feasible region as well as

the value of the objective function for each pair of Q_A and Q_B values (see slides 13 and 19 of the presentation of "Optimization").

- 4) Based on the previous analysis, what is the minimum cost of pumping for this industry?
- 5) What is the optimal water extraction Q_A and Q_B for the minimum cost of pumping?

Now, let's use the Open Solver model of excel to solve this problem. Use the file $Ex_5.xlsx$ - tab "4 Minimize" to create a linear optimization model. You can use as initial values for QA (cell G5) and QB (cell G6) 10 and 10 AF/year, respectively. Calculate *Cost A* in cell D8 using the formula provided above. Similarly, calculate *Cost B* in cell D11 using the formula provided above. Don't forget to link the objective function in cell D5 by summing the *Cost A* (Cell D8) plus *Cost B* (Cell D11). When creating the linear optimization model in Open Solver, don't forget to select "Minimize" instead of "Maximize" (Figure 17).

- 6) Turn in a screenshot of your linear model, similar to Figure 44.
- 7) What is the minimum cost of pumping (the optimum value of the objective function) obtained once the model has been run (cell D5)?
- 8) What are the optimal water extractions Q_A (cell G5) and Q_B (cell G6) for the minimum cost of pumping?
- 9) How do these results compare with the same analysis done before in questions 4 and 5 of this problem (see above)? Are these results similar?