University of California, Davis
Department of Land, Air and Water Resources

Optimization

ESM-121 Water Science and Management

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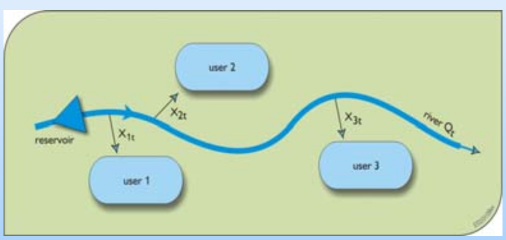
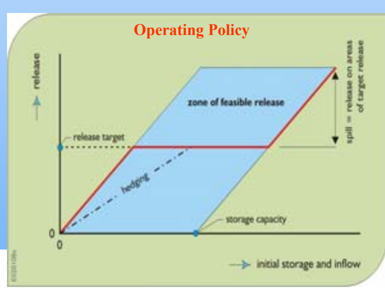
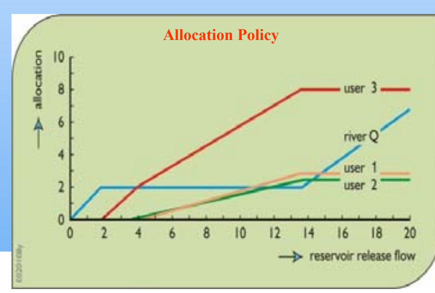
Presentation 5 of 6

NEED FOR TOOLS …

Allocate reservoir release R_t to 3 users and provide instream flow Q_t

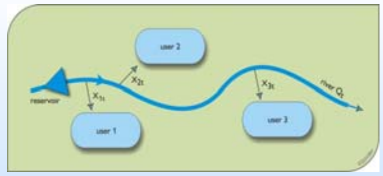


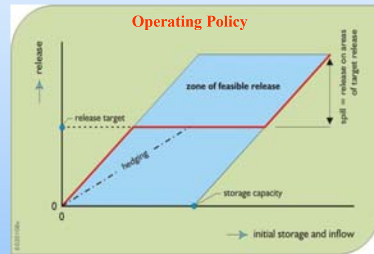


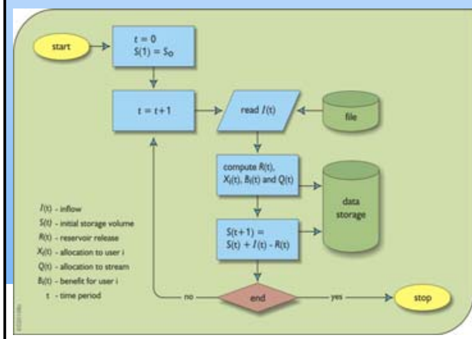


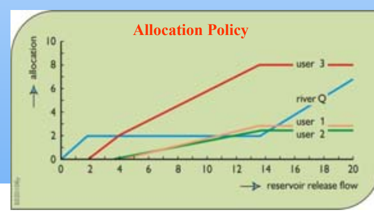
SIMULATION

- Address "**What if …**" questions
- What will likely happen
- Include larger hyd, econ, and env. data
- i.e. "evaluate change given a design or policy"



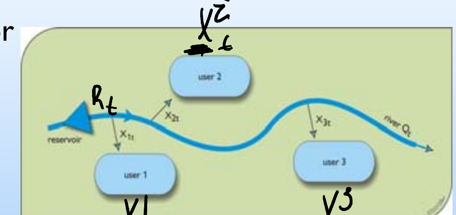






OPTIMIZATION

- Best answer:
  - "Maximize the Net Benefits …" or
  - "Minimize the shortages"
- Look for the best (ideal) operation
- Perfect foresight



Optimization model

Benefits: $B_i(x_{it})$

Decision Variables: x_{it}

Objective: $Maximize \sum_{t=1}^{T} \sum_{i=1}^{3} B_i(x_{it})$

Constraints:

$x_{1t} + x_{2t} + x_{3t} \leq R_t \quad t = 1, 2, \cdots$

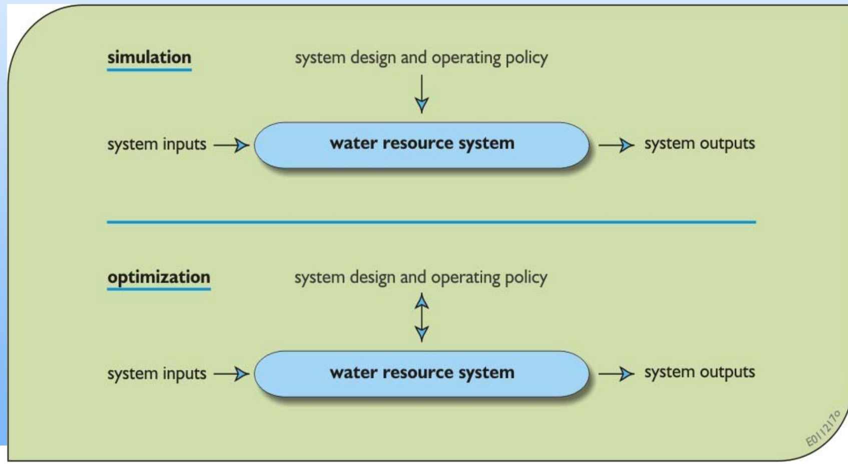
$S_{t+1} = S_t + I_t - R_t \quad t = 1, 2, \cdots$

$S_t \leq K \quad t = 1, 2, \cdots$

Optimization model

SIMULATION VS OPTIMIZATION

- Simulation models: Predict response to **given design**
- Optimization models: Identify **optimal design** or operation



OPTIMIZATION MODELS

OPTIMIZATION PROBLEMS

Find the decision variables, ***x***, that optimize (maximize or minimize) an objective function

$$\underset{\mathbf{x}}{\text{minimize}} \; f(\mathbf{x})$$

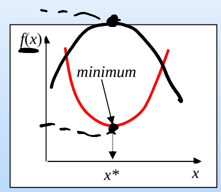
subject to

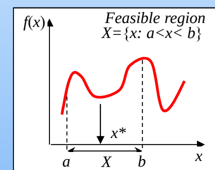
$$\mathbf{x} \in X$$
$$Ax + b \leq c$$
$$x \geq 0$$

Objective function

Decision variables

Constraint set





STEPS OF LINEAR PROGRAMING

1. Identify the Objective Function: **Maximize or Minimize**?
2. Define Objective Function (Write the Obj. Funct. Eq.)
3. Define the Constraints (Write the Constraints Eqs.)
4. Define the Feasible Region
5. Obtain the vertices of the feasible region
6. Substitute vertices in the Objective Function
7. Select the data where the value is Maximized/Minimized

EXAMPLE I

- Objective: **Maximize the Profits**
- Irrigation project
  - 1800 acre-feet of water per year

	Crop A	Crop B
Water requirement (Acre feet/acre)	3	2
Profit ($/acre)	300	500
Max area (acres)	400	600

- Decision variables
  - x_A = acres of Crop A to plant?
  - x_B = acres of Crop B to plant?

1,800 acre feet = 2,220,267 m[3]
400 acre = 1,618,742 m[2]

FEASIBLE REGION

$$\text{Maximize} Z = 300x_A + 500x_B$$

Subject to

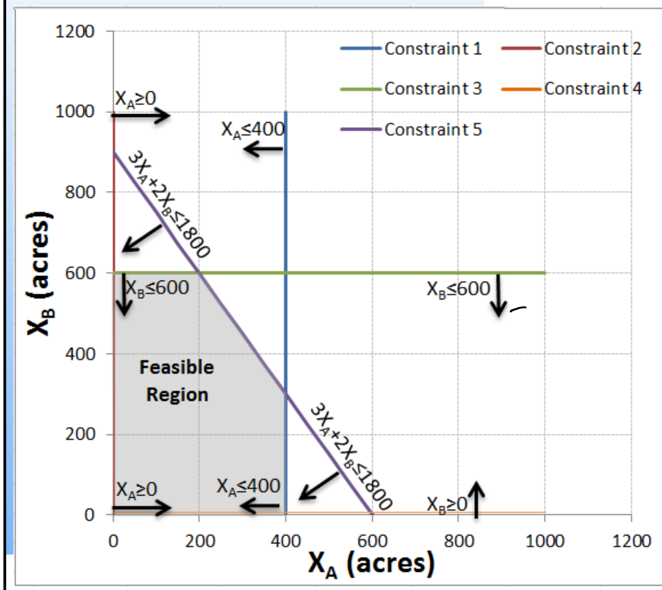
$$x_A \le 400$$
$$x_A \ge 0$$
$$x_B \le 600$$
$$x_B \ge 0$$
$$3x_A + 2x_B \le 1800$$

FEASIBLE REGION

$$\text{Maximize} Z = 300x_A + 500x_B$$

Subject to

$$x_A \le 400$$
$$x_A \ge 0$$
$$x_B \le 600$$
$$x_B \ge 0$$
$$3x_A + 2x_B \le 1800$$

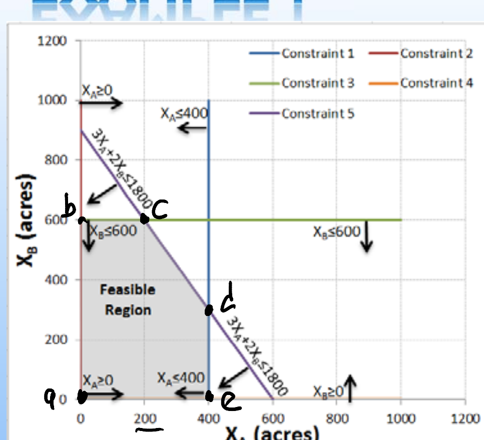


EXAMPLE I

$$\text{Maximize} Z = 300x_A + 500x_B$$

Subject to

$$x_A \le 400$$
$$x_A \ge 0$$
$$x_B \le 600$$
$$x_B \ge 0$$
$$3x_A + 2x_B \le 1800$$



	x_A	x_B
a	0	0
b	0	600
c	200	600
d	400	300
e	400	0

$3x_A + 2(600) = 1800$

$x_A = \frac{1800-1200}{3}$

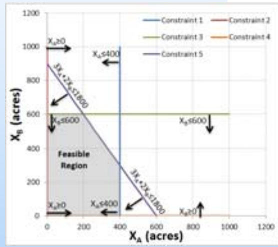
$x_A = \frac{600}{3} = 200$

$3(400) + 2(x_B) = 1800$

$x_B = \frac{1800-1200}{2}$

$x_B = \frac{600}{2} = 300$

EXAMPLE I



$$\text{Maximize} Z = 300x_A + 500x_B$$

Subject to

$x_A \leq 400$

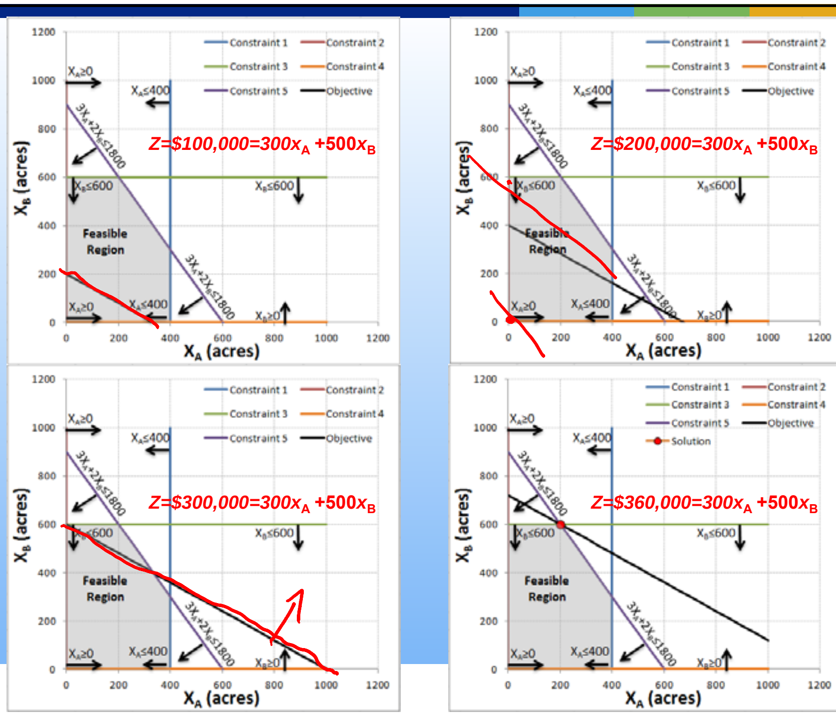
$x_A \geq 0$

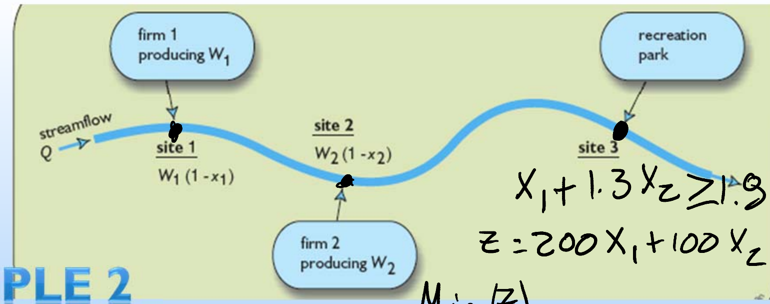
$x_B \leq 600$

$x_B \geq 0$

$3x_A + 2x_B \leq 1800$

Point	X_A	X_B	$Z=300X_A+500X_B$
a	0	0	300(0)+500(0)=0
b	0	600	300(0) + 500 (600) = 300,000
c	200	600	300(200) + 500(600) = 360,000
d	400	300	300(400) + 500(300) = 270,000
e	400	0	300(400) + 500 (0) = 120,000



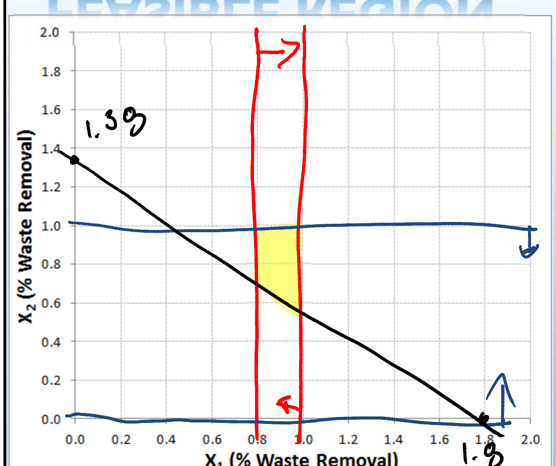


EXAMPLE 2

- Objective: **Minimize the Cost**
- Cost of 100% Waste Removal (WR)
  - C_1=\$200K
  - C_2=\$100K
- Decision variables
  - x_1 = % of WR at site 1; $0.8 \leq x_1 \leq 1.0$
  - x_2 = % of WR at site 2; $0 \leq x_2 \leq 1.0$
- $X_1 + 1.3x_2 \geq 1.8$

Parameter	Units	Value
Q_1	m^3/s	10
Q_2	m^3/s	12
Q_3	m^3/s	13
W_1	kg/day	250,000
W_2	kg/day	80,000
P_1	mg/l	32
P_2^{max}	mg/l	20
P_3^{max}	mg/l	20

FEASIBLE REGION



Minimize Z

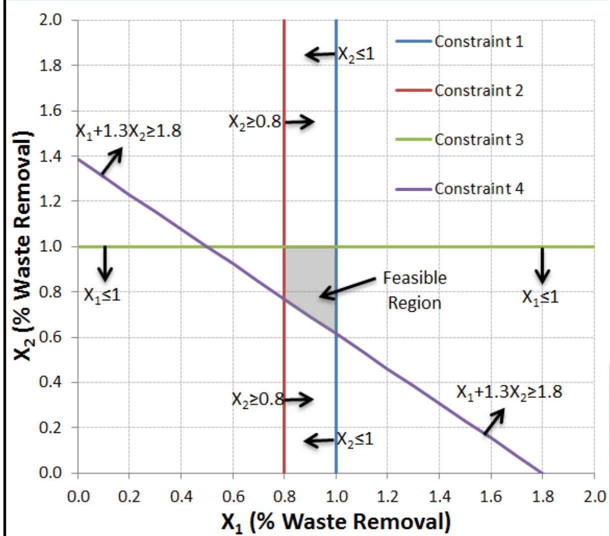
$$Z = C_1(x_1) + C_2(x_2)$$

$Where: C_1 = 200; C_2 = 100$

Subject to

$$x_1 \geq 0.8$$
$$x_1 \leq 1$$
$$x_2 \geq 0$$
$$x_2 \leq 1$$
$$x_1 + 1.3x_2 \geq 1.8$$

FEASIBLE REGION



$$\text{Minimize} C_1(x_1) + C_2(x_2)$$

$$Where: C_1 = 200; C_2 = 100$$

Subject to

$$x_1 \geq 0.8$$

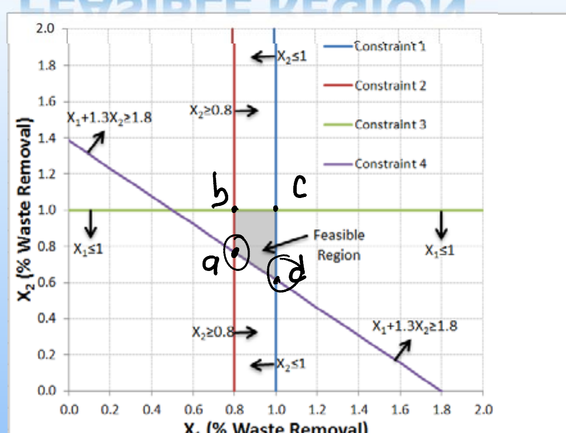
$$x_1 \leq 1$$

$$x_2 \geq 0$$

$$x_2 \leq 1$$

$$x_1 + 1.3x_2 \geq 1.8$$

FEASIBLE REGION



a (0.8, 0.78)

d (1.6, 0.62)

$$\text{Minimize} C_1(x_1) + C_2(x_2)$$

$$Where: C_1 = 200; C_2 = 100$$

Subject to

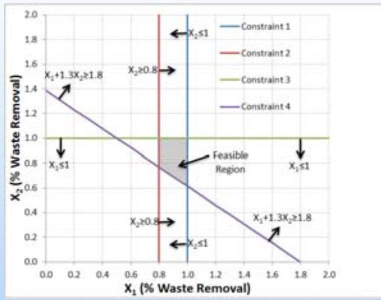
$$x_1 \geq 0.8$$

$$x_1 \leq 1$$

$$x_2 \geq 0$$

$$x_2 \leq 1$$

$$x_1 + 1.3x_2 \geq 1.8$$



200 x₁ + 300x₂

$$\text{Minimize} C_1(x_1) + C_2(x_2)$$

$$Where: C_1 = 200; C_2 = 100$$

Subject to

$$x_1 \geq 0.8$$

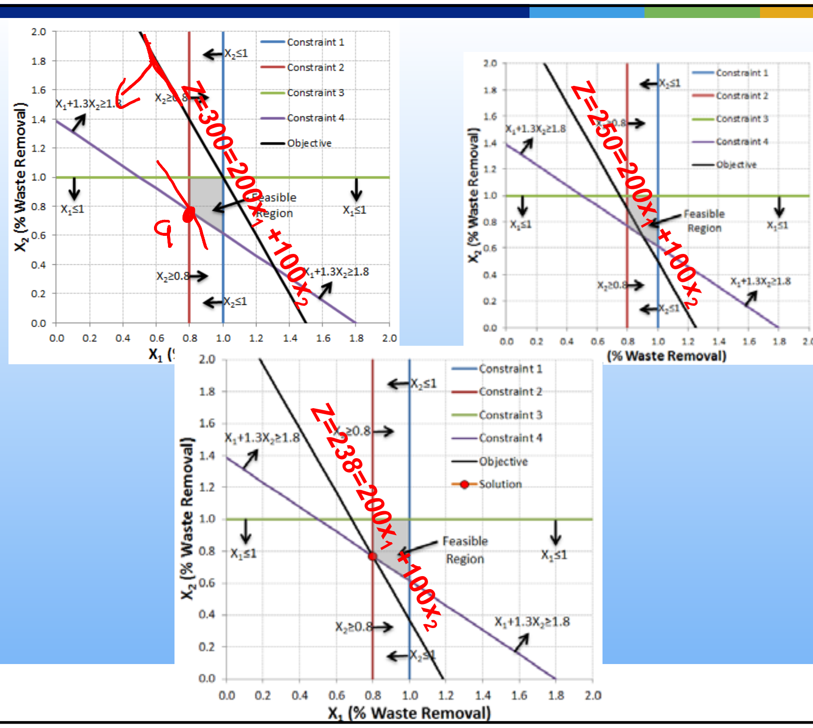
$$x_1 \leq 1$$

$$x_2 \geq 0$$

$$x_2 \leq 1$$

$$x_1 + 1.3x_2 \geq 1.8$$

Point	X_1	X_2	$Z=200X_1+100X_2$
a	0.8	0.78	200(0.8)+100(0.78) = 238
b	0.8	1.0	200(0.8)+100(1.00) = 260
c	1.0	1.0	= 300
d	1.0	0.62	= 262



UNITS

- 1 ft = 0.3048 m
- 1 m^3 = 28.3168x10 $^{-3}$ ft^3
- 1 m^3 = 35.3147 ft^3
- 1 ha = 10,000 m^2
- 1 acre = 43,560 ft^2
  = 0.4047 ha
  = 4047 m^2
- 1 gal = 3.785x10 $^{-3}$ m^3
  = 3.785 L

- 1 m^3 = 8.11x10^{-4} af
  10^9 m^3 = 8.11x10^5 af
  1 km^3 = 0.811 maf

- 1 m^3 = 264 gal
  10^9 m^3 = 264x10^9 gal
  1 km^3 = 264 bg
  1 km^3/yr = 0.7234 bgd