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RESEARCH PAPER

A New Approach for Parameter Estimation of Autoregressive Models Using Adaptive Network-Based Fuzzy Inference System (ANFIS)

Hamid R. Safavi[1] · Mohammad Hossein Golmohammadi[1] · Maryam Zekri[2] · Samuel Sandoval-Solis[3]

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Abstract Time series modeling plays an important role in different fields of science and engineering such as hydrology and water resources management. The proper estimation of the parameters in time series models is one of the essential steps of modeling. Yule–Walker, least square, Burge and forward–backward approaches are known, and common methods of parameter estimation are used in many time series studies. Recently, intelligent techniques such as adaptive network-based fuzzy inference system (ANFIS) have been used for time series modeling. Review of previous researches, especially in the field of hydrological time series, shows that these systems are often used as intelligent forecasting systems; indeed, they were considered as a black box. In this study, using ANFIS and its basic concepts, a new approach is devised for parameter estimation of autoregressive (AR) models. Performance of this approach is evaluated through the Akaike information criterion; also its application has been surveyed in time series forecasting by naturalized inflow of the Zayandehrud dam located in central Iran. Results show that the proposed approach has a good and effective performance for parameter estimation of AR models which can be depicted as a new "*intelligent approach.*" In addition, this capability of ANFIS in parameter estimation is a new application of ANFIS that was not addressed in the past. Also, the new driven method from ANFIS shows that this system can be employed as a parameter estimator for time series models such as AR models.

Keywords Parameter estimation · Autoregressive models · Hydrologic time series · Adaptive network-based fuzzy inference system (ANFIS) · Zayandehrud dam

1 Introduction

Modeling of hydrological processes resulting from the interaction of different variables is an important step in the water resources planning and management. Nonlinear and dynamic properties of hydrological processes and uncertainties of data are the main reasons of applying the time series modeling. On the other hand, time series models can be used for design and operation of water resource systems, according to the temporal and spatial statistics records by predicting hydrological variables such as river flow, rainfall, humidity and temperature. Time series analysis is one of the most common methods of forecasting and data generation of hydrological processes, especially for operation of water resource systems such as dams and rivers as surface water resources and aquifers as groundwater resources as well as conjunctive use systems (Safavi 2014). Research on hydrologic time series has been aimed at studying the main statistical characteristics, providing physical justification to some stochastic models, developing new and/or alternative models, improving the estimates of model parameters, developing new or improving existing modeling procedures, improving tests of goodness of fit, developing procedures on dealing with model and parameter uncertainties and studying the sensitivity of

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models and model parameters in applied hydrology (Salas et al. 1997).

In time series analysis and modeling, the relationships between inputs and outputs are mapped as a function of observed patterns in the past. Conventional time series methods including autoregressive (AR), autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models (Box and Jenkins 1976) have been used for hydrologic modeling. However, such models do not attempt to represent inherent nonlinear and dynamic characteristics of the hydrologic process and may not always perform well (Tokar and Johnson 1999; Nayak et al. 2004). Anyway, the conventional time series modeling methods have served the scientific community for a long time (Zounemat-Kermani and Teshnelab 2008). In time series modeling, it is necessary to determine parameters of each model to develop time series models (Sang 2012, 2013; Dutta et al. 2012). One of the most familiar methods of parameter estimation of AR models which is referred to as Yule–Walker (YW) method (Yule 1927; Walker 1931) is based directly on the linear relationship between the co-variances and the AR parameters (Stoica and Moses 2005; Hipel and McLeod 1994). Another method, the so-called least-squares (LSs) method is based on a least-squares solution of AR parameters using the time-domain equation (Stoica and Moses 2005). Burg (1975) expressed the problems of LS method and developed a new method for AR parameter estimation that is based on LS method to improve the mentioned problems and is depicted as Burg's method (Burg 1975). Details of these methods are provided in his PhD dissertation. Forward–backward (FB) method is another method that estimates AR parameters by minimizing the sum of a least-squares criterion for a forward model and the analogous criterion for a time-reversed model (Marple 1987). FB approach has the same order of computational complexity as the popular Burg algorithm. Marple (1987) concluded that the LS algorithm is an attractive alternative to the Burg algorithm for AR spectral estimation. The functions of these approaches can be used in some mathematics software such as MATLAB.

Recently, artificial intelligence techniques such as artificial neural networks (ANNs) and fuzzy logic have been used as efficient alternative tools for the modeling and forecasting of complex hydrologic systems and time series (Jeong et al. 2012; Kim and Singh 2013; Awan and Bae 2014). These methods are able to execute parallel computations and simulate nonlinear system which is hard to describe by traditional physical modeling. These intelligent systems have provided a wide range of applications in hydrology and water resources management (Safavi et al. 2013). The integration of neural networks and fuzzy logic has led to a new research area, namely neuro-fuzzy systems. Neuro-fuzzy systems have the potential to capture the benefits of both these fields in a single framework (Nayak et al. 2004). Adaptive network-based fuzzy inference system (ANFIS), which consists of the neural networks and fuzzy logic methods, has been used in many hydrologic applications such as rainfall-runoff process for predicting daily runoff at multiple gauging stations (Nourani and Komasi 2013) and improving rainfall forecasting efficiency (Akrami et al. 2013), reservoir operation (Valizadeh and El-Shafie 2013), decision support systems (Petrovic et al. 2006), discharge routing (Khatibi et al. 2011), evapotranspiration estimation (Cobaner 2011), river streamflow and dams inflow forecasting (Sanikhani and Kisi 2012; El-Shafie et al. 2007), and water demand forecasting (Tabesh and Dini 2009). ANFIS eliminates the basic problem of fuzzy systems design (obtaining a set of fuzzy if–then rules) using the learning capability of an ANN, effectively, for automatic fuzzy if–then rule generation and parameter optimization.

Aforementioned researches are examples of many researches which have investigated the applications of ANFIS in hydrologic filed, especially in time series modeling and forecasting. In these researches, ANFIS is considered as a black box which means that after training and testing, it can be used as an intelligent model to simulate or predict the uncertain future. So, it was not used to estimate various parameters using inner parameters of ANFIS such as weights and output of membership functions of fuzzy inference system (FIS). In this study, based on the basic concepts of ANFIS, a new approach for parameter estimation of AR models is devised which is a novel technique for estimating parameters of time series such as AR time series models. Based on the hybrid method for training of ANFIS, the new approach can be considered as a combination of LS, FB and Burg's approaches with emphasis that the new approach is derived from ANFIS and it is a new capability of this system. Performance of the models developed by new approach in prediction is surveyed by mean squared relative error (MSRE), the coefficient of efficiency (CE) and mean absolute error (MAE) in comparison of prediction with models developed by YW, LS, Burg and FB approaches. On the other hand, presented approach shows a new application of ANFIS. The applicability and performance of this approach have been surveyed by the Zayandehrud dam inflow as case study.

2 Case Study: Zayandehrud Dam

The Zayandehrud River located in Gavkhooni basin is a vitally important river for agricultural development, domestic water supply and economic activity of the Isfahan

Province in west-central Iran. The Zayandehrud basin has covered about 63% of the Gavkhooni basin. It is a completely closed basin having no outlet to the sea. The river is about 350 km long and runs in a roughly west–east direction, originating in the Zagros Mountains, west of the city of Isfahan, and terminating in the Gavkhooni wetland to the east of the city. The area of the Zayandehrud basin is about 41,524 km^2, which includes about 2.5% of the total area of Iran. About 4200 km^2 of area of the Zayandehrud basin is allocated to the Zayandehrud dam basin (Safavi et al. 2013). Figure 1 shows the situation of the Gavkhooni basin in Iran and also the sub-basins and main branches of the Zayandehrud dam basin.

As shown in Fig. 2, the Zayandehrud River originating in the Zardkooh-Bakhtiari Mountain and then jointed to the Pelasjan and Samandegan rivers constitutes the main branches of the Zayandehrud; these are inflows of the Zayandehrud dam. The streamflow of the main branch of the Zaynbdehrood is measured at the Ghale Shahrokh station. Also, the streamflows of the Pelasjan and Samandegan rivers are measured at the Eskandari and Menderjan stations, respectively. Two diversion tunnels in operation since 1957 can deliver 540 MCM of water from Karun basin annually, known as Koohrang tunnels, while a third tunnel, expected to be ready in a few years, will deliver a further 250 MCM of water annually (Murray-Rust et al. 2000). Cheshmeh-Langan tunnel with annual capacity of 164 MCM was established in 2008 to deliver water from the Dez basin to the Pelasjan River in the Zayandehrud basin.

Figure 2 shows the observed data of the tunnels and rivers in the Zayandehrud dam basin.

In this study, naturalized data of the mean annual inflow of the Zayandehrud dam are used for developing the AR models. Naturalized flows are calculated to represent historical streamflow in a river basin in the absence of human development and water use (Danner et al. 2006). To naturalize the Zayandehrud dam inflow, discharge of diversion tunnels has been removed from total inflows of dam and water allocations along the rivers have been added. Based on reports of IWRM researches in the Zayandehrud basin (IWRM in Isfahan 2014), the total allocation along the rivers is about 4.09 m^3/s in the upstream of the Zayandehrud dam. Table 1 shows the mean annual naturalized inflow of the Zayandehrud dam for the water years from 1980 to 2012.

Skewness of this series is about 0.612, and skewness with significance level of 0.1 for normalized time series is 0.637. So this series can be considered as a normal series with confidence level of 90%. The Hurst coefficient is about 0.638, so based on Hurst phenomenon, this series has adequate memory for modeling with dynamically persistent. More details about Hurst phenomenon can be found in Sakalauskienė (2003). Downtrend of the series is removed, and series becomes stationary. After preliminary checks, this series is used to develop AR models employing the new technique and previous approaches.

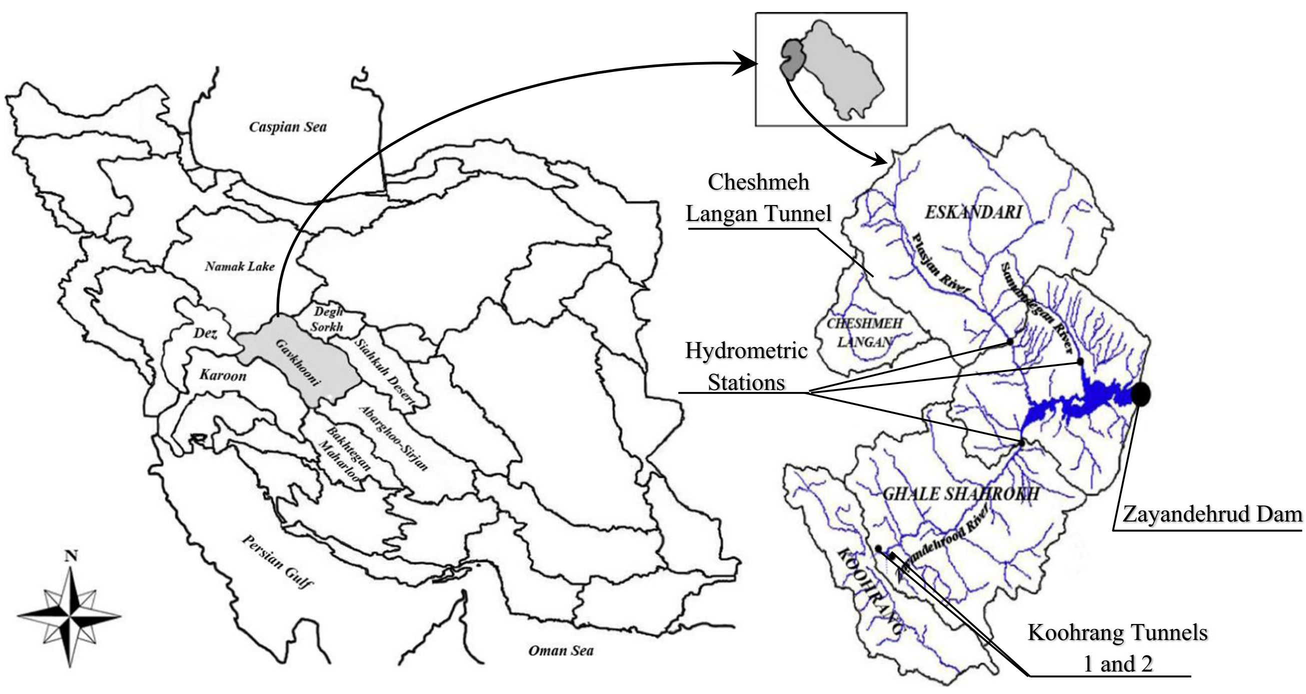


Fig. 1 Situation of the Gavkhooni basin and the Zayandehrud dam basin in Iran

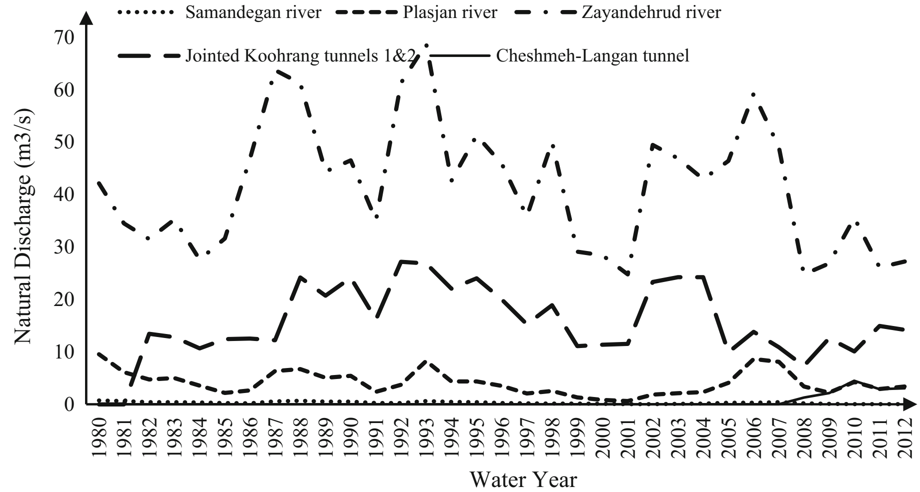


Fig. 2 Streamflow of rivers and amount of transfer water from other basins to the Zayandehrud dam basin

Table 1 Mean annual naturalized inflow (MANI) of the Zayandehrud dam for the years 1980 to 2012 (m^3/s)

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
MANI	60.89	47.28	26.58	32.14	24.20	23.36	37.09	60.44	48.89	33.73	29.80
Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
MANI	23.76	42.82	56.73	28.92	36.19	33.42	26.53	37.38	22.78	21.27	15.88
Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
MANI	30.33	27.60	26.91	45.65	57.68	50.13	21.86	16.59	28.33	14.80	13.53

3 Materials and Methods

3.1 Hydrologic Time Series Model

Box and Jenkins (1976) have done extensive researches on time series modeling in the past half century. In 1970, they developed the AR model to analyze historical data that had relations within itself. They presented the general and main steps of time series modeling as: (1) selection of model type, (2) identification of model form, (3) estimation of model parameters and (4) diagnostic check of the model (Salas et al. 1997).

AR model is a very common model in time series modeling (Yevjevich 1972). For short data records, the AR method yields reasonable estimate (Marple 1980). The notation AR(p) indicates an autoregressive model of order p. Whenever x_1, x_2, …, x_n are variables of a time series, AR models can be presented as follows (Box and Jenkins 1976):

$$\mathrm{AR}(p):\ Z_t = \sum_{i=1}^{p} \Phi_i Z_{t-i} + \varepsilon_t \tag{1}$$

where Z_t is the standard series with normal distribution, p is order of the model, Φ_1, Φ_1, …, Φ_p are the model parameters, and ε_t is the residual of the series with mean zero and variance σ_ε^2. It is an independent series with normal distribution. Also, Z_t is defined as:

$$Z_t = \frac{x_t - \mu}{\sigma} \tag{2}$$

where μ and σ are mean and variance of time series, respectively.

The method which is used to estimate the autoregressive model parameters affects the performance of the AR model. There are numerous techniques to estimate AR parameters from data samples. Four methods of autoregressive-parameter estimation from data samples are considered here: the Yule–Walker approach (YW) explained by Marple (1980), the least-squares approach (LS) explained by Farebrother (1988), Legendre (Stigler 1981), Gauss (Sprott 1978) and Adrich (Aldrich 1998), Burg's method explained by Burg (1975) and forward–backward (FB) approach explained by Marple (1980, 1987).

Yule–Walker is one of the first and known approaches for parameter estimation of time series models. De Hoon et al. (1996) showed that the Yule–Walker method should not be used as an estimator of parameters of AR models if the auto-covariance matrix is poorly conditioned. They concluded that in this case, the relatively small covariance estimate bias can lead to a large deviation in the estimated parameters, resulting in an invalid model. Their investigations indicated that the least-squares approach and Burg's method are still able to estimate the autoregressive model correctly. Least squares should be used with caution though, as it does not guarantee the estimated autoregressive model to be stable, as a result a small deviation in the parameter estimates may cause the estimated poles to move outside the unit circle. In this case, the estimated autoregressive model will be invalid. Finally they introduced Burg's method as the most reliable estimation technique, as it provides reliable parameter estimates as well as an estimated model guaranteed to be stable (see Burg 1975).

FB approach was not included in De Hoon's researches. Marple (1980) investigated the problems of LS and Burg's approaches and suggested a new approach named Forward–Backward approach. He introduced a new recursive algorithm based on the LS solution for the AR parameters using forward and backward linear prediction.

In this study, all of the approaches are used to estimate the AR parameters to compare the results of them with the results of the new proposed approach (ANFIS approach). YW approach is not used for prediction by developing time series models because of the results of De Hoon et al. (1996).

3.2 Time Series Prediction by AR Models

To utilize time series for prediction, first of all, the best model should be selected. A mathematical formulation which considers the principle of parsimony in model building is the Akaike information criterion (AIC) proposed by Akaike (1974). AIC is a measure of the relative quality of a statistical model, for a given set of data. Also, AIC provides a means for model selection. AIC was presented to check which order is more adequate than other orders of the fitted model. The AIC for an AR(p) model is formulated as (Hu 2007):

$$\mathrm{AIC}(p) = n\ln\left(\hat{\sigma}_{\varepsilon}^{2}\right) + 2p \tag{3}$$

where $\hat{\sigma}_{\varepsilon}^{2}$ is the estimated residual variance of fitted model and n is the sample size. The "best" model is the one with minimum AIC value.

As the samples x_t cannot be predicted exactly, a residue is introduced, which is defined as the difference between the observed value and the estimated value (De Hoon et al. 1996):

$$\text{residue} \equiv x_t - \hat{x}_t = \varepsilon_t \tag{4}$$

where x_t and $\hat{x}_t$ are observed value and estimated value by dependence model.

After selecting the best model by AIC, it is used for forecasting. The forecasted values for $Z_{t+l};\ l \geq 1$ for an origin at t with lead time l are written as (Salas et al. 1997):

$$Z_t(l) = \Phi_1 Z_t(l-1) + \cdots + \Phi_p Z_t(l-p) \tag{5}$$

Obviously, in prediction by AR models, the past residues (ε_t) do not affect the forecast calculations. Thus, for $0 < \Phi_1 < 1$, an AR(1) model will give $Z_t(l) < Z_t$. So, in this study, for a wise comparison, models are used for two step (2 year) prediction.

3.3 Adaptive Network-Based Fuzzy Inference Systems (ANFIS)

Fuzzy logic and fuzzy sets theory first introduced by Zadeh (1965) which is imposed to describe language and human reasoning in the context of mathematics (Firat et al. 2009). There are two types of widely used fuzzy inference systems, Takagi–Sugeno FIS and Mamdani FIS (Jang et al. 1997). The most important difference between these systems is the definition of consequent parameters (Takagi and Sugeno 1985).

The permanent growing interest in intelligent technology merging, particularly in merging of neural and fuzzy technology, the two technologies that complement each other, to create neuro-fuzzy or fuzzy-neural structures, has largely extended the capabilities of both technologies in hybrid intelligent systems (Bezdek 1993). The advantages of neural networks in learning and adaptation and those of fuzzy logic systems in dealing with the issues of human-like reasoning on a linguistic level, transparency and interpretability of the generated model, and handling of uncertain or imprecise data, enabled one of the higher level intelligent systems depicted as "adaptive network-based fuzzy inference system (ANFIS) to be built (Zadeh 1965; Hornik et al. 1989; Wang 1997; Palit and Popovic 2005).

ANFIS is a multi-layer adaptive network-based fuzzy inference system initially developed by Jang (1993) and later on widely applied in engineering (Jang and Sun 1995). The general structure of the ANFIS is presented in Fig. 3. Selection of the FIS is the major concern when designing an ANFIS to model a specific target system. The ANFIS system used in this study is Sugeno type of FIS. The corresponding equivalent ANFIS architecture is presented in Fig. 3b, where nodes of the same layer have similar functions.

A Sugeno system by two inputs and one output can be expressed by two rules as:

Rule 1: *if* x *is* A_1 *and* y *is* B_1, *Then* $f = p_1x + q_1y + r_1$
Rule 2: *if* x *is* A_2 *and* y *is* B_2, *Then* $f = p_2x + q_2y + r_2$

The functioning of the ANFIS is as follows (Jang et al. 1997):

Layer 1 Each node in this layer generates membership grades of an input variable. The node output OP_i^1 is defined by:

$$\mathrm{OP}_i^1 = \mu_{A_i}(x) \text{ for } i = 1,2 \text{ or } \mathrm{OP}_i^1 = \mu_{B_{i-2}}(y) \text{ for } i = 3,4 \tag{6}$$

where, x (or y) is the input to the node; A_i (or B_{i-2}) is a fuzzy set associated with this node, characterized by the

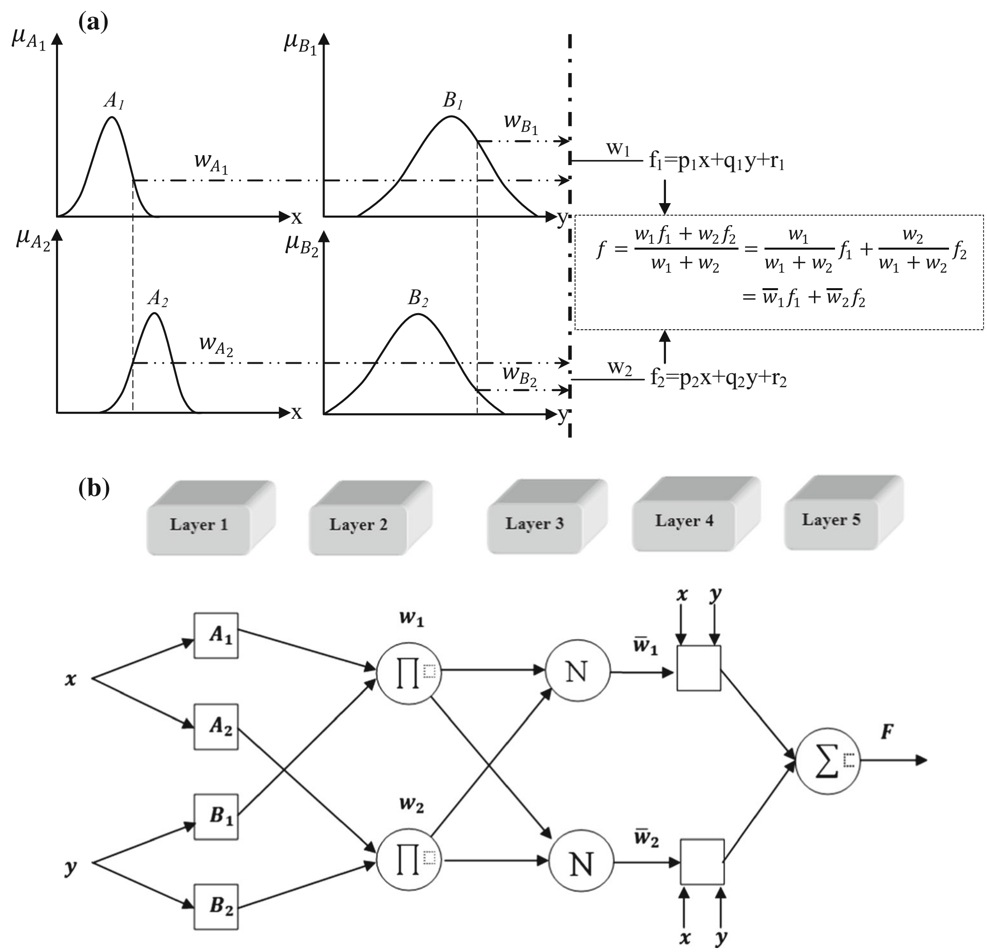


Fig. 3 **a** Fuzzy inference system. **b** Equivalent ANFIS architecture (Nayak et al. 2004; Jang et al. 1997)

shape of the membership functions (MFs) in this node and can be any appropriate functions that are continuous and piecewise differentiable such as Gaussian, bell-shaped, trapezoidal shaped, or triangular-shaped functions. Assuming a bell-shaped function as the MF, the output OP_i^1 can be computed as:

$$\mathrm{OP}_i^1 = \mu_{A_i}(x) = \frac{1}{1+\left(\frac{x-c_i}{a_i}\right)^{2b_i}} \tag{7}$$

where $\{a_i, b_i, c_i\}$ is the parameter set that changes the shape of the MF with a maximum equal to 1 and an infimum equal to 0.

Layer 2 Every node in this layer multiplies the incoming signals, denoted by π, and the output OP_i^2 that represents the firing strength of a rule computed as:

$$\mathrm{OP}_i^2 = w_i = \mu_{A_i}(x) \times \mu_{B_i}(y); \quad i = 1,2 \tag{8}$$

where w_i is the activation weight.

Layer 3 The ith node of this layer, labeled as N, computes the normalized firing strengths as:

$$\mathrm{OP}_i^3 = \bar{w}_i = \frac{w_i}{w_1 + w_2}; \quad i = 1,2 \tag{9}$$

Layer 4 Node i in this layer computes the contribution of the ith rule toward the model output, with the following node function:

$$\mathrm{OP}_i^4 = \bar{w}_i f_i = \bar{w}_i(p_i x + q_i y + r_i) \tag{10}$$

where $\bar{w}_i$ is the output of layer 3 and ith rule, and $\{p_i, q_i, r_i\}$ is the parameter set.

Layer 5 The single node in this layer computes the overall output of the ANFIS as:

$$\mathrm{OP}_i^5 = \text{Overal Output} = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \tag{11}$$

Training of these systems means that by using training data, nonlinear parameters of layer 1 and linear parameters of layer 4 are set, so that for each input the desired output is achieved. Hybrid-learning algorithm is one of the most important methods of training ANFIS. In this method, for training parameters in the layer 1 and layer 4, back propagation (BP) and least square error (LSE) methods are used, respectively. Details of the algorithm and mathematical background can be found in Jang and Sun (1995).

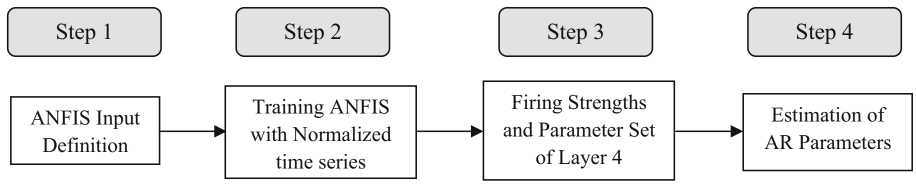


Fig. 4 Methodology framework to estimate parameters of AR models

Here, it is significant that there is a great similarity between the hybrid method of training in ANFIS and some parameters approaches such as LS and FB approach for AR models. This paper has followed how this feature of ANFIS can be used to estimate parameters of the AR models.

3.4 Methodology

In this study, based on the basic concepts of time series modeling and ANFIS, a new approach is provided to estimate parameters of AR models. Figure 4 shows the steps of the new proposed approach to estimate parameters of AR models.

Step 1 For estimating parameters of each AR model, inputs and outputs of the system must be defined according to their equations. For estimation of parameter of AR models, Z_{t-i}, $i = 1, 2, \ldots, p$ and Z_t are considered as the input and output of ANFIS, respectively. This idea is derived from comparing the structure of ANFIS and its output equation (Eq. 11) with respect to AR model equation (Eq. 1). With change in input and output of ANFIS, various types of AR models can be achieved. For example, Figs. 5 and 6 show the proposed structures of ANFIS for AR(1) and AR(2) models.

Step 2 Training developed ANFIS model is the second step. In this step, ANFIS can be trained by back propagation, least-squares estimation and hybrid methods.

Step 3 After training ANFIS, firing strengths of layer 3 should be extracted from the model and normalized; also parameter sets of layer 4 (p_i, q_i, r_i) should be extracted from trained ANFIS model.

Step 4 The combination of normalized firing strengths (Eq. 9) and parameter set of layer 4 will provide estimation parameters of the relevant AR model.

Here for instance, how parameters of AR(1) and AR(2) can be estimated in steps 3 and 4 is explained.

According to Fig. 5, and Eq. (10) and (11), the output of ANFIS can be obtained for AR(1) model, as follows:

$$\begin{aligned} Z_t &= \bar{w}_1 f_1 + \bar{w}_2 f_2 + \cdots + \bar{w}_n f_n \\ &= \bar{w}_1(p_1 Z_{t-1} + r_1) + \bar{w}_2(p_2 Z_{t-1} + r_2) + \cdots \\ &\quad + \bar{w}_n(p_n Z_{t-1} + r_n) \end{aligned} \tag{12}$$

This equation can be rewritten as:

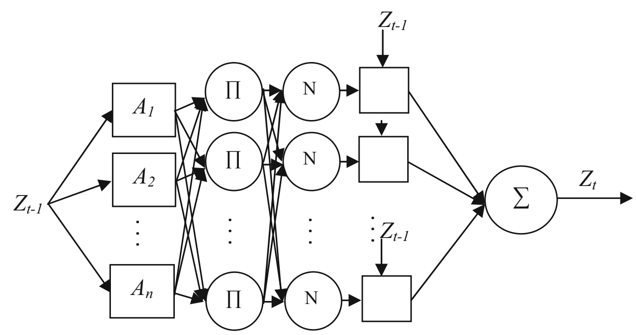


Fig. 5 Structure of ANFIS for parameter estimation of AR(1)

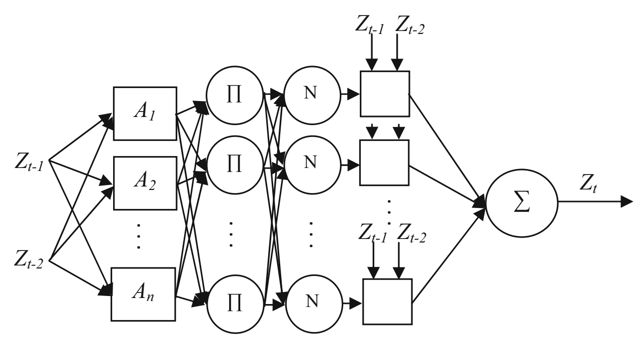


Fig. 6 Structure of ANFIS for parameter estimation of AR(2)

$$\begin{aligned} Z_t = {} & (\bar{w}_1 p_1 + \bar{w}_2 p_2 + \cdots + \bar{w}_n p_n) Z_{t-1} \\ & + (\bar{w}_1 r_1 + \bar{w}_2 r_2 + \cdots + \bar{w}_n r_n) \end{aligned} \tag{13}$$

Comparison between Eq. (1) and (13) leads to estimation of Φ_1 for AR(1) using ANFIS according to structure of Fig. 2, as follows:

$$\Phi_1 = \bar{w}_1 p_1 + \bar{w}_2 p_2 + \cdots + \bar{w}_n p_n \tag{14}$$

So, according to Fig. 6 and Eq. (10) and (11), the output of ANFIS can be obtained for AR(2) model, as follows:

$$\begin{aligned} Z_t &= \bar{w}_1 f_1 + \bar{w}_2 f_2 + \cdots + \bar{w}_n f_n \\ &= \bar{w}_1(p_1 Z_{t-1} + q_1 Z_{t-2} + r_1) \\ &\quad + \bar{w}_2(p_2 Z_{t-1} + q_2 Z_{t-2} + r_2) \\ &\quad + \cdots + \bar{w}_n(p_n Z_{t-1} + q_n Z_{t-2} + r_n) \\ &\Rightarrow Z_t = (\bar{w}_1 p_1 + \bar{w}_2 p_2 + \cdots + \bar{w}_n p_n) Z_{t-1} \\ &\quad + (\bar{w}_1 q_1 + \bar{w}_2 q_2 + \cdots + \bar{w}_n q_n) Z_{t-2} \\ &\quad + (\bar{w}_1 r_1 + \bar{w}_2 r_2 + \cdots + \bar{w}_n r_n) \end{aligned} \tag{15}$$

Comparison between Eq. (1) and (15) leads to estimation of Φ_1 and Φ_2 of AR(2) using ANFIS according to structure of Fig. 3, as:

$$\begin{aligned}\Phi_1 &= \bar{w}_1 p_1 + \bar{w}_2 p_2 + \cdots + \bar{w}_n p_n \\ \Phi_2 &= \bar{w}_1 q_1 + \bar{w}_2 q_2 + \cdots + \bar{w}_n q_n\end{aligned} \tag{16}$$

Same methodology can be used for estimation parameters of various higher orders of AR models such as AR(3), AR(4) and so on. It is important that among all intelligent systems just ANFIS whose structure is able to adopt to various time series models such as AR. It is obvious that conformity of ANFIS structure and output with equations of AR models is a unique capability of ANFIS which leads to the new proposed idea.

Proposed approach is surveyed by a series which is explained in Sect. 2. This series is annual data of the Zayandehrud dam inflow for a 33-year period from 1980 to 2012. Thirty-one years of data are used to estimate parameters of AR models, and two last years (2011–2012) are used to evaluate these models to predict the Zayandehrud dam inflows. It is important that in the presented approach, the weights and parameters of third and fourth layers of ANFIS models obtained after training are used to estimate the parameters of AR models; so it is not necessary to make training and testing collections, separately. In other words, based on the new intelligent technique, AR parameters are estimated without any test or validation processes; just with the best trained ANFIS systems, AR parameters are estimated using the proposed approach. In this study, after training the networks, activation weights and linear parameters of fourth layer of ANFIS are obtained using MATLAB. Then, parameters of five AR(p), $p = 1, \ldots, 5$ models are determined using the new proposed approach (ANFIS) and compared with parameters which are estimated by other common approaches such as Yule–Walker (YW), least square (LS), Burg and forward–backward (FB) approaches. For all of the proposed systems, three bell membership functions are considered for each input. Also, all of the systems are trained in 1000 epochs.

To assess and compare the performance of these parameters, developed models with these parameters are assayed in forecasting. First, to select the best model, AIC is calculated for all models with previous and proposed approaches using Eq. 3. Anyway, the best models selected by Akaike criteria are used for the next step, called time series prediction. Three goodness-of-fit criteria such as the mean squared relative error (MSRE), coefficient of efficiency (CE) and mean absolute error (MAE) are utilized to evaluate the performances of previous and proposed approaches in forecasting. CE ranges from $-\infty$ at the worst case to $+1$ for a perfect correlation. The CE statistic provides a measure of the ability of a model to predict inflows which are different from the mean (Nash and Sutcliffe 1970). According to Shamseldin (1997), a CE of 0.9 and above is very satisfactory, 0.8–0.9 represents a fairly good model, and below 0.8 is deemed unsatisfactory. MSRE provides a more balanced perspective of the goodness of fit at moderate inflows (Karunanithi et al. 1994). MAE, which computes all deviations from the original data regardless of sign, is not weighted toward high flow events. MSRE and MAE range from 0 for a perfect condition to $+\infty$ at the worst case (Dawson and Wilby 2001). They are defined as:

$$\mathrm{MSRE} = \frac{\sum_{i=1}^{n} \frac{(x_i - \hat{x}_i)^2}{x_i^2}}{n} \tag{17}$$

$$\mathrm{CE} = 1 - \frac{\sum_{i=1}^{n} (x_i - \hat{x}_i)^2}{\sum_{i=1}^{n} (x_i - \bar{x}_i)^2} \tag{18}$$

$$\mathrm{MAE} = \frac{1}{n} \sum_{i=1}^{n} |x_i - \hat{x}_i| \tag{19}$$

where $\hat{x}_i$ and x_i are predicted/estimated and observed values under investigation, respectively; $\bar{x}$ is the mean or average of the observed values; and n is the number of total values.

Based on (De Hoon et al. 1996), the YW approach is not suitable for autoregressive modeling and forecasting, so in this study it is not applied for forecasting of AR models.

4 Results and Discussion

To compare results of proposed approach (ANFIS approach) and other approaches, the average annual inflow of the Zayandehrud dam (see Table 1) has been used to develop AR models. Table 2 shows the results of proposed method (ANFIS) and other approaches for AR(p), $p = 1, \ldots, 5$.

Comparison of the estimated parameters in all AR models shows that the new approach has a reasonable performance in parameter estimation. Parameters derived from ANFIS approach are very close to least square, Burg's and forward–backward methods, especially least square method. It is because for training ANFIS, parameters of layer 4 are trained by method of least square error which is a method very close to the least square (LS) approach. Also, for training parameters of first layer of ANFIS, back propagation method is used which is very close to Forward–Backward (FB) approach. Based on proposed technique in this study, parameters are dependent on weights and parameters obtained from the fourth layer of ANFIS models. This is the reason of the proximity of the proposed approach and LS approach. Also, dependence of

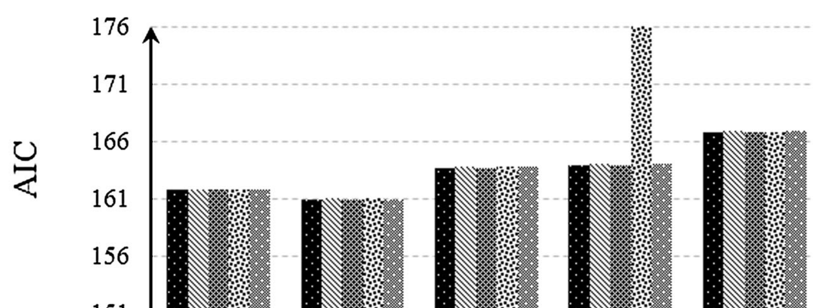
Table 2 Results of parameter estimation of the AR models using proposed intelligent approach (ANFIS) and YW, LS, Burg and FB approaches for the Zayandehrud dam inflow

Model	Parameters	Approach				
		Yule–Walker	Least square	Burg's lattice based	Forward–backward	ANFIS
AR1	Φ_1	0.4251	0.4560	0.4714	0.4714	0.4602
AR2	Φ_1	0.5567	0.5973	0.6308	0.6014	0.5924
	Φ_2	−0.3096	−0.3310	−0.3381	−0.3389	−0.3231
AR3	Φ_1	0.5672	0.6069	0.6356	0.6076	0.6044
	Φ_2	−0.3284	−0.3035	−0.347	−0.3315	−0.3032
	Φ_3	0.0339	0.01446	0.01399	0.01457	0.0195
AR4	Φ_1	0.5762	0.6058	0.6393	0.6015	0.6052
	Φ_2	−0.4155	−0.4331	−0.4395	−0.4676	−0.4310
	Φ_3	0.1843	0.1455	0.1835	0.1298	0.1434
	Φ_4	−0.2651	−0.2779	0.2668	−0.2734	−0.2719
AR5	Φ_1	0.5879	0.6148	0.6346	0.5788	0.6151
	Φ_2	−0.4236	−0.4484	−0.4363	−0.4632	−0.4473
	Φ_3	0.2026	0.1492	0.1758	0.1080	0.1495
	Φ_4	−0.2905	−0.2301	−0.2556	−0.2545	−0.2298
	Φ_5	0.0442	−0.0224	−0.0176	−0.0278	−0.0191

parameters of last layer of ANFIS to the first layer is the reason of concordance of the proposed method with FB method. Due to Burg's approach using the least square method in its algorithm, the results of this approach are very close to LS and ANFIS methods. So in general, because of structure and theoretic characteristics of ANFIS, the new approach has properties and features of LS, FB and Burg's approaches.

To select the best model, the results of calculated AIC are shown in Fig. 6. It is clear that the fewer AICs in all approaches belong to the AR(2). Although the Akaike criterion is used to select the best order of model, it can be used to evaluate the performance of new proposed approach against other approaches. Obviously, as shown in Fig. 7 the less AICs in all AR models belong to the ANFIS approach. This means the new approach has less residual variance against other approaches and in comparison has a good performance in parameter estimation of the AR models.

Regarding to the results of Fig. 7, LS and FB approaches have less AICs after ANFIS approach; it may be because of the training methods of ANFIS (BP, LSE and hybrid method) are very close to these approaches (FB and LS). So it can be said that ANFIS is a combination of the previous methods; also, it is a new intelligent approach to estimate parameters of different time series models. It should be emphasized that in this paper it is not claimed that the new provided approach is better than others. But it has a good performance in comparison with others. Here a new "Intelligent" approach is devised by ANFIS. This



	AR1	AR2	AR3	AR4	AR5
ANFIS	161.720	160.927	163.697	163.916	166.704
Yule-Walker	161.776	160.973	163.816	164.070	166.952
Least Square	161.721	160.929	163.699	163.918	166.704
Burg	161.725	160.980	163.785	176.114	166.791
Forward Backward	161.725	160.937	163.735	164.005	166.910

Fig. 7 Results of AIC for all models and approaches

Table 3 Comparison of approaches for forecasting of AR(2) models developing by estimated parameters

Criterion	Approach			
	Least square	Burg	Forward–backward	ANFIS
MSRE	2.964	3.193	3.034	2.906
CE	0.999	0.999	0.999	0.999
MAE	0.668	0.669	0.668	0.669

approach can be considered as a new approach for parameter estimation of time series models; it can also be considered as one of the new performances of ANFIS.

Anyway, AR (2) is used to predict inflows of the Zayandehrud dam for the years 2011–2012. Table 3 shows the results of three goodness-of-fit criteria to compare the approaches for forecasting of AR(2) models developed by parameters estimated by common approaches and ANFIS approach (Safavi et al. 2015).

It is clear that the results of proposed approach (ANFIS) are acceptable and better compared with results of other approaches in time series prediction. Results of CE show that the new approach has a good performance, the same as other approaches. MSRE of predictions shows that the ANFIS approach has a better performance than other approaches. This means the model developed by parameters which are estimated by ANFIS approach has more balanced perspective of the goodness of fit in prediction of annual inflows to the Zayandehrud dam. Also, the low MAE belongs to the ANFIS and FB approaches, so the predictions by these approaches have less deviation from the observed annual inflows. Therefore, in general, the proposed approach (ANFIS) can be successfully applied for parameter estimation of AR models, as an intelligence approach.

5 Conclusion

In this study, adaptive network-based fuzzy inference system (ANFIS) was applied to estimate the parameters of autoregressive (AR) models. By developing various structures of ANFIS with regard to the different AR models, this study presented a new idea for parameter estimation of these models. To evaluate the performance of proposed method, the simulations were assayed for the Zayandehrud dam inflows. Results of parameter estimation by new approach were compared with parameters estimated by other common approaches such as Yule–Walker (YW), least square (LS), Burg and forward–backward (FB). Also, developed AR models were utilized to predict the inflows of the Zayandehrud dam using estimated parameters by these approaches. Results show that the new approach (ANFIS) can be used as a useful technique for estimation of parameters of AR models. By using the proposed technique in this study, parameters of AR models can be obtained only with changing inputs and outputs for each model. This approach can be used as a new and intelligent approach to estimate the parameters of AR models. This approach also introduces another capability of ANFIS, previously known as black box, using the basic concepts of it.

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