

Reliability Based Optimum Reservoir Design by Hybrid ACO-LP Algorithm

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Abstract Optimal design of irrigation and water supply reservoirs under reliability constraints may be categorized as large combinatorial optimization problems. In this paper, the reliability based optimum design of a single water supply reservoir is formulated as a mixed integer programming and a hybrid algorithm is introduced for its solution. To eliminate iterative procedures in reliability-based reservoir design and operation, the reliability requirements are directly embedded into the modeling framework and treated as different sets of constraints. Adaptive penalty method is used for constraint handling in the solution methodology. The proposed algorithm couples an ant colony optimization (ACO) optimizer with a virtual linear programming (LP) model for the solution of the resulted NP-hard mixed integer nonlinear programming problem. Dez reservoir for irrigation water supply with 480 months of inflow is used to demonstrate the method and its performance. The structure and solution methodology is verified by the solution to the inverse problem. It is shown that the proposed hybrid model can efficiently solve the problem for various combinations of reliability measures in a multiple period modeling scheme. It is illustrated that under some circumstances and specific reliability values, the mixed integer nonlinear programming (MINLP) solver may even fail to address a feasible and local optimal solution. Although operating rule is not included in the operational scheme, the procedure is capable of identifying coefficients for decision rules with any proposed structure.

Keywords Reliability · Reservoir design · ACO-LP · Hybrid algorithm · Reservoir operation · Dez Reservoir

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1 Introduction

Reservoir analysis problem may be classified into (1) capacity determination, (2) long-term operations planning, and (3) real time operation. Generally speaking, it has a multitude of decision nature ranging from determination of optimum reservoir storage capacity for planning purposes to deriving optimal reservoir operating policies for real time operation. Although the general concepts (i.e., the relationship between inflow, performance criteria decision and state variables such, reservoir storage and release, etc....) for planning, design and/or operation remains the same, the decision variables, objective functions, constraints and the structure of models may vary for different types of reservoir problems (Simonovic 1992). Considering the random nature of reservoir inflows, determination of minimum reservoir capacity for satisfying a given demand with an acceptable level of reliability is often referred to as reservoir sizing problem. It addresses one of the main problems in reservoir analysis. It determines the minimum reservoir capacity required to meet a given demand with an acceptable level of reliability.

Since their introduction by ReVelle et al. (1969), integration of linear decision rules (LDR's) with chance-constrained reservoir operating models have extensively been used for *direct* determination of reservoir capacity and operation rule to meet specified releases and storage volume reliability targets (Simonovic and Marino 1980 and 1982; Houck et al. 1980; Afshar and Marino 1990; Afshar et al. 1991; Malekmohammadi 1998; Satishkumar et al. 2010). However, the chance-constrained programming approach, as an alternative to stochastic programming that does include reliability in the optimization, has been criticized and reported to generate very conservative solutions (Hogan et al. 1981). Criticizing the existing chance-constraints reliability based models in optimizing reservoir designs, Strycharczyk and Stedinger (1987) claimed that the reliability programming formulation of the reservoir management problem employs a very restrictive operating policy. In a numerical example the reliability programming model's constraints overestimated reservoir capacity requirements by an order of magnitude. In a good documentary paper, Simonovic (1992) presented a reservoir simulation-optimization model (RESER) which makes use of a direct search technique for finding the minimum required capacity in two levels; namely optimization and simulation levels. Minimum reservoir capacity for meeting water demands and reliability criteria are determined and checked in optimization and simulation levels, respectively. If the desired reliability is not met, the reservoir capacity is increased by a step size. The trial and error procedure continues until the reliability constraints are satisfied. Without any doubt the proposed "double-reliability" scheme has an advantage over other methods used for determining reservoir capacity. In a fairly recent study, Afzali et al. (2008) developed and successfully applied a multi-reservoir reliability-based simulation model that uses an iterative single-period linear programming (LP) in each time step to minimize the hydropower design parameters for achieving a desired reliability in hydropower generation.

Reliability-based optimum reservoir design is a NP-hard Mixed-Integer Nonlinear programming (MINLP) problem with both real and integer variables. Traditional methods (such as branch-and-bound) for optimal solution of an NP-hard MINLP problem may fail to converge to feasible solution and often requires long computation time, particularly for large number of integer variables.

To relax the drawbacks of common MINLP solvers and realizing the computational advantages of hybrid models in dealing with large NP-hard MINLP problems (Afshar et al. 2009a, b; Reis et al. 2005 and 2006); this paper presents a hybrid (ACO-LP) model for direct minimization of reservoir capacity with reliability constraints. Reliability constraints are embedded into the modeling framework to eliminate iterative procedures in reliability-based

reservoir design and operation. Although operating rule is not included in the present modeling scheme, the proposed procedure may easily be re-formulated to identify rules coefficients in a reservoir operation problem. Both, the mathematical formulation and the solution procedure, may easily be expanded for reliability-based hydropower reservoir design. Using Dez water supply reservoir as a case example, the paper demonstrates the method and its performance in a large multi-period problem. Historical inflows for 480 months are used to obtain the minimum capacity under various pre-specified reliability levels. The structure of the model and solution methodology is validated by the solution to the inverse problem. To compare and verify performance of the proposed approach, results are compared with those of a branch and bound MINLP solver and checked against those achieved using another hybrid model.

1.1 Reliability-Based Optimization of Reservoir Design

In a deterministic design optimization, the designs are often bonded with the design constraints, leaving little or no latitude for uncertainties and chance of failure. Robust design optimization and reliability based design optimization are among the methodologies that may be used to address the uncertainties in designs.

Robust designs are designs at which the variation in the performance functions is kept minimal. Reliable designs, on the other hand, keep the chance of failure of the system within pre-specified low values (Agarwal 2004). Reliability based optimum design (RBOD) deals with obtaining optimal design characterized by a predefined probability of failure. In RBOD problems, there is a trade-off between obtaining higher reliability and lowering cost, if justified. In a RBOD formulation, the critical failure modes in deterministic optimization are replaced with a single or set of constraints on probabilities of failures. Generally speaking, RBOD intends to optimize a merit function while satisfying set of reliability constraints. The reliability constraints address the probability of failure corresponding to each of the failure modes of the system or a single constraint on the system probability of failure (Agarwal 2004).

Reservoir reliability is defined as the probability that the reservoir will perform the required function, i.e., provide the outflow required to satisfy the water demand, at a specified period of time under stated conditions. In accordance with engineering standards of care, reservoirs are to be designed to provide stability and durability. The reservoir design criteria are not intended to establish any particular design approach, but rather to ensure water system adequacy, reliability, and compatibility with existing and future conditions.

The reliability of a system is the probability of the system's successful performance in the specific period of time under determined conditions (Chow et al. 1988). Based on this definition the reliability (α) of an event ω with the probability of $P[\omega]$ may mathematically be presented as.

$$\alpha := P[L(t) < C(t); t \in II] =: 1 - \beta \quad (1)$$

Where β is the probability of failure, $L(t)$ and $C(t)$ are the loading and capacity, respectively, II is the planning time horizon.

Using the reliability concept as defined, the reservoir design problem may be formulated as a reliability-based optimization model in which the reservoir capacity may be minimized for a given set of performance reliabilities over the operation horizon. For irrigation water supply reservoirs it is common to set two types of reliabilities on releases. The first constrain may commit the operator to satisfy the full demand with a predefined reliability level (α). For the periods of failure, in which the demand has not been fully satisfied, the shortage (deficit) may not exceed a pre-specified fraction of the total demand (β). In another word, the release may

never be smaller than a fraction of demand specified by (γ). Realizing the stochasticity of the inflow to the reservoir, the problem may be formulated as a mixed integer optimization problem with a NP-hard structure.

The objective function of the problem in this research is the minimization of the reservoir capacity under desired reliability.

$$\text{Minimize}(Cap) \quad (2)$$

Subject to the following constraints:

$$S_{t+1} = S_t + I_t - E_t - RK_t - RS_t \quad \forall t = 1, 2, \dots, T \quad (3)$$

$$S_{\min} \leq S_t \leq Cap \quad \forall t = 1, 2, \dots, T \quad (4)$$

$$RS_t \geq RE \quad \forall t = 1, 2, \dots, T \quad (5)$$

$$RK_t \geq Z_t \times demk_t \quad \forall t = 1, 2, \dots, T \quad (6)$$

$$RK_t \leq demk_t \quad \forall t = 1, 2, \dots, T \quad (7)$$

$$RK_t \geq \gamma \times demk_t \quad \forall t = 1, 2, \dots, T \quad (8)$$

$$Area_t = a \times (S_t + S_{t+1})/2 + b \quad \forall t = 1, 2, \dots, T \quad (9)$$

$$E_t = he_t \times (Area_t + Area_{t+1})/2 \quad \forall t = 1, 2, \dots, T \quad (10)$$

$$Rel = \left(\sum_{t=1}^T Z_t \right) / \left(\sum_{t=1}^T t \right) \geq \alpha \quad \forall t = 1, 2, \dots, T \quad (11)$$

$$\left(\sum_{t=1}^T demk_t - RK_t \right) / \left(\sum_{t=1}^T demk_t \right) \leq \beta \quad \forall t = 1, 2, \dots, T \quad (12)$$

$$Z_t = \begin{cases} 1 & \text{if demand is fully satisfied} \\ 0 & \text{otherwise} \end{cases} \quad \forall t = 1, 2, \dots, T \quad (13)$$

Where Cap is the maximum reservoir storage (decision variable), S_t is the reservoir storage, I_t is the inflow to the reservoir, E_t is the evaporation from reservoir, RK_t is the release from reservoir for agricultural demand, RS_t is the release to downstream as spill and environmental demand, $Area_t$ is the reservoir area, he_t is the net monthly evaporation height from the reservoir, RE is the minimum required environmental flow, $demk_t$ is the agricultural demand, Rel is the reliability, α is the reliability index and β is the normalized deficit index, and t stands for period. One may note that in formulation of Eq. 3, both the spill from the reservoir and the environmental flow are included in RS_t , since both are released to the downstream. Constraint defined by Eq. 5 satisfies the environmental flow released to downstream. Constraints number 6 and 7 force the release to be equal to the demand for the periods in which $Z = 1$. For the periods with $Z = 0$, in which the demand may not totally be satisfied, agricultural release shall not be less than a pre-specified fraction (γ) of the demand. This restriction is taken care of with constraint number 8 in which the agricultural release will never fall below that limit. The reliability and volumetric deficit constraints for the entire planning horizon are considered in

Eqs. 11 and 12, respectively. Within the maximum and minimum operation levels, the reservoir surface area is assumed to be a linear function of the storage volume (Eq. 9). Based on the existing data, the coefficients are estimated as $a=0.0157$ and $b=11.291$.

The reliability based optimum design problem addressed by Eqs. 2–13 is a mixed integer nonlinear programming (MINLP) model with large number of integer variables (Z). In fact the number of integer variables equals to the number of operational periods if only one water user with defined reliability level is assumed. The number of integer variables will double if another user with different target reliability is imposed on the system. In this research we presented an ACO-LP for solution of the proposed reliability based optimum design of the reservoir.

2 Proposed Hybrid Decomposition ACO-LP Algorithm

Application of hybrid algorithms for optimum design and/or operation of water resources problems have a fairly recent origin (Cai et al. 2001; Reis et al. 2005, 2006). Combination of metaheuristic and/or evolutionary algorithms with a LP model for decomposition and two stage solutions of highly non-linear and large scale water resources problems have been quite satisfactory (Afshar et al. 2010). The basic concept with the proposed hybrid algorithm is to decompose a large-scale nonlinear mixed integer optimization problem between a search-based ACO sub-module and a large-scale LP sub-module for handling the complicating and non-complicating variables, respectively. In this study the reservoir capacity and integer variables are identified as set of complicating variables.

The inter relation between the two sub-modules for solution of the reliability-based reservoir design problem is presented in Fig. 1. In the proposed approach, the original problem is decomposed into two sub-modules. The first sub-module employs ACO algorithm as an efficient solver to minimize the reservoir capacity (original objective function). To account for the nonfeasibility of the trial solutions generated with the ACO algorithm, the objective function is penalized with the total constraints violations. The total constraint violation is minimized in the second sub-module. The second sub-module is a virtual LP model with the same general structure of the original NLP model. It aims to minimize the total constraints' violations for the trial values of the decision variables as generated and tested in the first sub-module. In other words, the generated solutions for complicating variables in the optimization search-based ACO solver is imported into the virtual LP model to minimize the constraints' violations. The weighted sum of the constraints violation from the virtual LP model is used to penalize the objective function in the original reliability based design model. The proposed approach may mathematically be presented as:

$$\text{Minimize : Capacity} + w \times G \quad (14)$$

Where G =vector of constraints' violations minimized in virtual LP model and w =positive penalty assigned for constraints' violations. Penalizing the original objective function for constraint violation will facilitate the process in addressing the set of feasible solutions.

The virtual LP model borrows its general structure from the original MINLP design model with the objective of minimizing the sum of non-feasibilities for the trial values for complicating variables generated with the ACO solver. The sum of infeasibilities

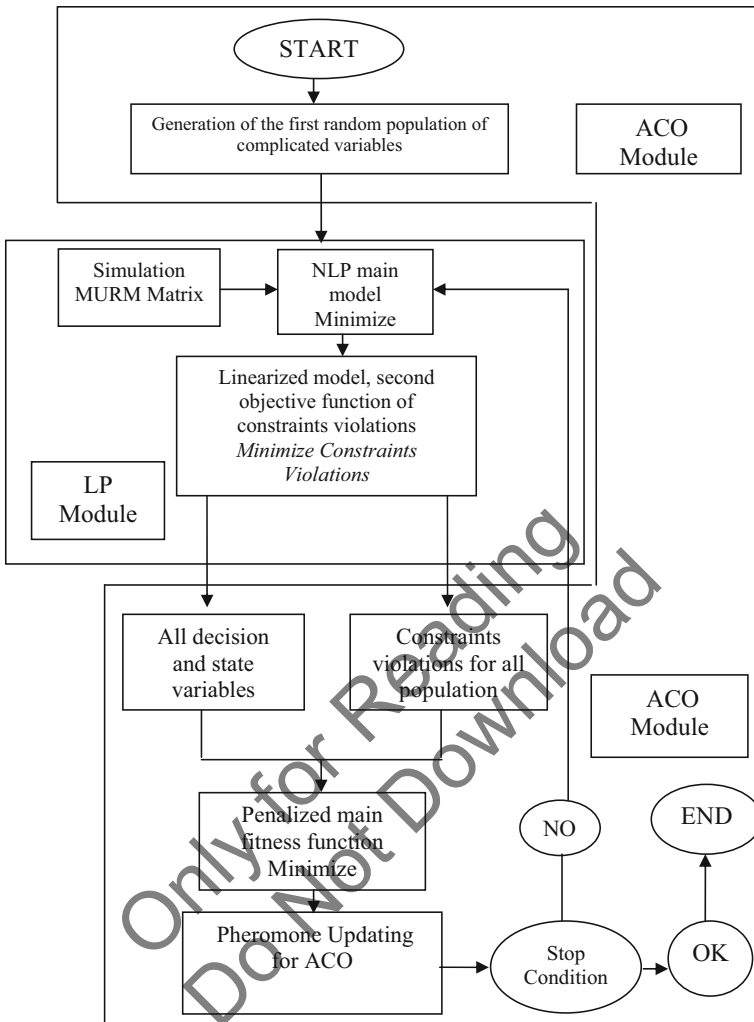


Fig. 1 Descriptive flow chart of the general interrelation between the two sub-modules

is defined as sum of the absolute values of deviations from the constraints. Thus, the virtual LP model may be formulated as:

$$\text{Minimize } G = \sum_{i=1}^m CSV_i \forall CSV_i \geq 0 \tag{15}$$

Subject to:

$$f_i(x) = CSV_i \quad \forall i = 1, \dots, m \tag{16}$$

Where m =total number of the constraints; and CSV_i =constraint violation for the i th constraint. In this proble, CSV_i refers to constraints 11 and 12 in original problem. Determination of the value of penalty coefficient has always been a challenge in the use of meta-heuristic models to solve the constrained problems. Large penalty coefficients often reduce the exploration by

concentrating on limited search area and accelerating the convergence to a premature solution and vice versa. Afshar (2008) and Afshar et al. (2009a, b) presented a penalty adapting ant algorithm to eliminate the dependency of ant algorithms on the penalty parameter used for the solution of constrained optimization problems. The method uses an adapting mechanism for determination of the penalty parameter leading to elimination of the costly process of penalty parameter tuning. This paper employs the same iterative penalty method by modifying the penalty coefficient at any iteration as.

$$g_i = \varphi \left(\frac{f_i}{f_{best}} + 1 \right) \quad \forall \quad i = 1, \dots, m \tag{17}$$

Where g_i is the value of the penalty coefficient to be used at the next i th iteration, φ is a tunable constant parameter, f_i is the objective function in iteration number i and f_{best} is the best objective function until the i th iteration.

Employing the penalized objective function (Eq. 14), the reliability based optimization model will be solved by minimizing Eq. 14 subject to the constraints number 3–10. In this formulation, the reliability constraints defined earlier by Eqs. 11–12 are now satisfied by the objective function of virtual LP (Eq. 15). Note that the virtual LP sub-problem is formulated to minimize the total constrains violation with values of complicating variables recommended by the ACO solver which aims at minimizing the penalized objective function.

The proposed structure for the hybrid algorithm follows the structure proposed by Afshar et al. (2009a, b) and slightly differs from that recommended by Cai et al. (2001). In their work, the linearized model with the trial values of complicating variables estimates the constraints' violations and objective function values in one-step procedure.

To handle the reliability constrains we employed the penalty approach by penalizing the objective function. Rearranging the constraints number 11 and 12 along with introducing the CSV as the violation from the reliability constraints we have:

$$CSV_1 = \left(\frac{\sum_{i=1}^t z_i}{t} - \alpha \right) \tag{18}$$

$$CSV_2 = \left(\frac{\sum_{i=1}^t demk_t - \sum_{i=1}^t RK_t}{\sum_{i=1}^t demk_t} - \beta \right) \tag{19}$$

Although the Branch and Bound algorithm is a powerful one, in some cases with large number of integer decision variables and very restricted feasible zone, it may fail.

The proposed hybrid algorithm starts with generating the initial trial solutions to define the initial path in the ACO solver. The reservoir capacity and integer variables are identified as set of complicating variables. For this 480-period reliability based optimum design problem, 481 complicating variables is identified which includes 480 integer values for Z_r . The lower and

upper bounds on the reservoir capacity is defined and a random generator is employed to generate the initial reservoir capacity and binary values for Z_t in the entire periods. The trial solutions with assumed values of the complicating variables are then imported into the main MINLP design model to relax the nonlinearities and give it a linear structure, named virtual LP model. The virtual LP model is then solved to minimize a measure of total infeasibilities evaluated as sum of deviations from the constraints. Solution to the virtual LP model will be imported into the ACO solver to evaluate the fitness of the trial solutions as penalized objective function. New solutions identified for the complicating decision variables in the ACO solver will be used to reform the virtual LP for the new trial values of the decision variables. This process continues until the sum of infeasibilities approaches zero and other termination criteria are met (Fig. 1).

3 Ant Colony Optimization (ACO); An Overview

Ant Colony Optimization is one of the most popular and efficient meta-heuristic algorithms solving large scale optimization problems especially when the decision variables are in discrete forms. Various versions of ACO in discrete and continuous domains, as well as single or multiple objective structures, have been used to derive optimal design and operating policies for various water resources problems (Abbaspour et al. 2001; Maier et al. 2003; Afshar et al. 2009a, b). Kumar and Reddy (2006) compared the performance of ACO algorithm with real coded GA to derive operating policies for a multi-purpose reservoir system. They emphasized superior performance of ACO, especially in longtime horizon operation models. Although ACO algorithms were originally proposed for discrete search spaces, it has successfully been applied to continuous domains in water resources problems (Jalali et al. 2007; Madadgar and Afshar 2009). In ACO algorithms, the optimization search procedure is made by the number of artificial ants. Each ant builds a solution, or a component of it, starting from an initial state selected according to some problem dependent criteria. While building its own solution, each ant collects information on the problem characteristics and on its own performance, and uses this information to modify the representation of the problem, as seen by the other ants. In an ACO algorithm, the decision policy can be expressed as (Dorigo et al. 1996).

$$p_{i,j}(t) = \begin{cases} \frac{\tau_{i,j}(t) [\eta_{i,j}]^\beta}{\sum_{l:(i,l) \in \theta_i} \tau_{i,l}(t) [\eta_{i,l}]^\beta} & q > q_0 \\ I \operatorname{argmax}_{i,l \in \theta_i} \left(\tau_{i,j}(t) [\eta_{i,j}]^\beta \right) \{(i,j)\} & \text{otherwise} \end{cases} \quad (20)$$

In Eq. 20, $q \sim \text{uniform}[0, 1]$ and q_0 is the exploration–exploitation factor ($0 \leq q_0 \leq 1$). If $q > q_0$ in Eq. 20, the decision is governed by the same probabilistic decision equation as used in the original ant system (AS) which $\alpha=1$ and if $q \leq q_0$, the edge with the highest $\tau_{i,j}(t) [\eta_{i,j}]^\beta$ value and zero probability is given to all other edges). For $q \leq q_0$ exploitation is encouraged as the edge that has received the greatest amount of pheromone addition will generally be selected, and for $q > q_0$ exploration of alternate edges is encouraged. Exploration receives higher emphasis as $q_0 \rightarrow 1$. In ant colony system

(ACS), the pheromone is locally and globally updated. In “local” updating, the pheromone on a selected edge by an ant is directly updated after it has generated its solution. This degradation discourages the re-selection of edges within iteration and works to balance the exploitative decision policy by further encouraging exploration of alternate edges. The operation to locally update the pheromone of edge (i, j) selected by an ant is.

$$\tau_{i,j}(t) \leftarrow \rho_l \tau_{i,j}(t) + (1 - \rho_l) \tau_0 \tag{21}$$

In Eq. 21, ρ_l is the local pheromone decay coefficient, and τ_0 is the initial pheromone intensity laid on all edges. ACS also involves global updating (i.e., updating the pheromone paths at the end of iteration when all m ants have generated their solutions). The global updating rule in ACS is the same to that in AS, but in ACS only the path with the global-best solution receives additional pheromone. The global updating rule is given by.

$$\tau_{i,j}(t) = \rho_g \tau_{i,j}(t) + (1 - \rho_g) \Delta \tau_{i,j}^{gb}(t) \tag{22}$$

In Eq. 22, ρ_g is the global pheromone decay coefficient and $\Delta \tau_{i,j}^{gb}(t)$ is given by.

$$\Delta \tau_{i,j}^{gb}(t) = \frac{Q}{f(S^{gb}(t))} I_{S^{gb}(t)}\{(i, j)\} \tag{23}$$

In Eq. 23, $S^{gb}(t)$ is the global best path found up to iteration t . The updating rule given in Eq. 18 acts as an encouragement for exploitation, as only the best solution is reinforced with additional pheromone.



Fig. 2 Dez reservoir location in southwest Iran

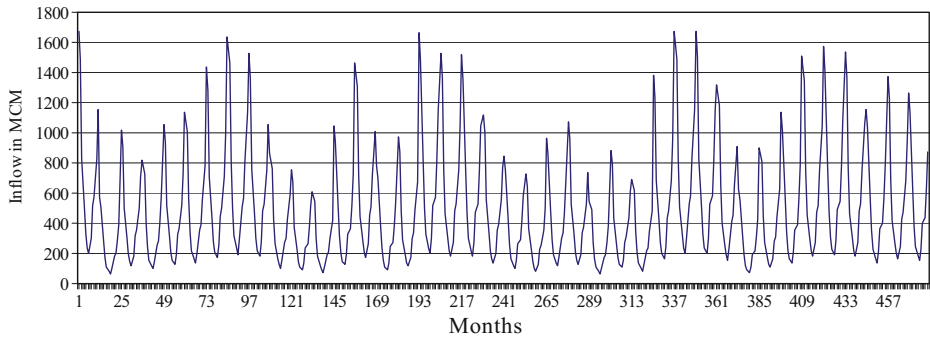


Fig. 3 Monthly inflow to the reservoir

4 Application of the Model

4.1 Problem Setting and Case Study

To illustrate the performance of the proposed hybrid algorithm in solving reliability-based optimum reservoir design problems, the Dez reservoir in southern Iran was selected as a case study (Fig. 2). Relatively large standard deviation of the seasonal and total annual inflow to the reservoir makes the design a reliability sensitive one and a good example for illustration of the proposed approach. Monthly historical inflow to the reservoir for a 40-year period is presented in Fig. 3. Table 1 presents the monthly agricultural demand for downstream irrigation use. Average annual inflow to the reservoir and annual demand are estimated as 8500 and 5900 MCM, respectively. The Dez reservoir is already under operation with total storage capacity, dead storage, and effective storage as 3340, 830, and 2510 MCM, respectively.

For partial verification of the model and its results, the performance of the model is double checked with the inverse solution method in which the reliability (α) was maximized for a fixed reservoir capacity. In the inverse model the constraints remain the same as those of the original model. As expected, the same results were obtained for the reliability. To validate the model its performance for large number of cases are checked against those of classical MINLP and another hybride model. Its similar performance in highly diverse water resources problems and their statistical analysis and comparison remains to be tested.

5 Results and Discussion

The proposed ACO-LP hybrid model is used to find the set of optimum values of the Dez reservoir capacity which are required to satisfy the specified agricultural demand under various reliability and volumetric deficit measures. The problem was solved for design periods of 240, and 480 periods under different reliability and volumetric deficit measures.

Table 1 Monthly irrigation demand (MCM)

Month	May	Jun	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb	Mar	Apr
Irrigation demand	516.4	603.7	757.2	831.1	818.8	706	467.6	318	163	150.1	203	365.5

Table 2 Dez reservoir capacities (in 1000MCM) under different α and β and $\gamma=0.7$ for 240 month period

Model	$\alpha=0.70$		$\alpha=0.75$		$\alpha=0.80$		$\alpha=0.85$		$\alpha=0.9$	
	MILP	ACO-LP	MILP	ACO-LP	MILP	ACO-LP	MILP	ACO-LP	MILP	ACO-LP
$\beta=0.2$	5.725	6.373	7.255	8.054	8.264	8.523	9.825	10.435	11.869	12.746
$\beta=0.25$	4.223	4.629	5.542	6.161	6.092	6.455	7.587	8.195	–	9.258
$\beta=0.30$	3.582	4.150	4.040	4.491	5.472	6.057	6.261	6.963	–	8.30
$\beta=0.35$	3.170	3.410	3.594	3.912	4.031	4.510	5.341	5.904	–	6.82
$\beta=0.40$	2.799	3.003	–	3.716	–	4.213	–	5.777	–	6.006
$\beta=0.45$	2.491	2.609	–	3.211	–	4.113	–	5.208	–	5.318
$\beta=0.50$	2.116	2.584	–	2.900	–	3.800	–	4.700	–	5.068

For validation purpose and without intending to make a detail comparison and solid conclusion, the results of the ACO-LP and the MINLP for a 240-period problem are presented in Table 2. As expected, the MINLP solves the problem in relatively shorter time, provided that a feasible solution is found. It is observed that the merits of the MINLP model diminishes as the feasible zone narrows down by imposing higher limitations on reliability and β . As illustrated, for some combinations of the reliability index and the maximum permissible deficit in agricultural demand (β), the MINLP fails to locate a feasible solution. However, if a feasible solution is found, the MINLP solver dominates the ACO-LP in computational time and the quality of the results. Employing 40 years monthly inflow data, the verified ACO-LP model was used to derive the optimal reservoir capacity for reliabilities ranging from 70 to 90 % for various values of maximum permissible deficit in agricultural demand (β) and different monthly water shortages (γ). For $\gamma=0.5$ and different values of α and β and 15,000 function evaluations, results of the both ACO-LP and GA-LP hybrid models are presented in Table 3. Although there are slight differences between the results for different combinations of α , β and γ , it is difficult to strictly distinguish between their performances and merits. Similar performance and results of ACO-LP and GA-LP may also be used as a measure of validation. Slightly better performance from ACO-LP might be attributed to a unique feature of incremental solution building mechanism in the ACO algorithm. This unique feature is quite beneficial for constraint handling and trial feasible solution development. Results for other values of maximum monthly water shortage measure, γ , and different combinations of α and β for the same number of function evaluations are presented in Tables 4, 5 and 6. For most of the cases the rate of increase or decrease in reservoir capacity is almost uniform with varying values of α , β and γ . In some case, however, the rate of increase or decrease is quite

Table 3 Dez reservoir capacities (in 1000MCM) under different α and β and $\gamma=0.5$ for 480 month period

Model	$\alpha=0.70$		$\alpha=0.75$		$\alpha=0.80$		$\alpha=0.85$		$\alpha=0.9$	
	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP
Function evaluation	15,000	15,000	15,000	15,000	15,000	15,000	15,000	15,000	15,000	15,000
$\beta=0.30$	2.151	2.131	2.402	2.326	2.980	2.898	4.696	4.643	7.883	7.610
$\beta=0.25$	2.151	2.131	2.402	2.326	2.980	2.898	4.696	4.643	7.883	7.610
$\beta=0.20$	2.151	2.131	2.402	2.326	2.980	2.898	4.696	4.643	7.883	7.610
$\beta=0.15$	2.866	2.836	2.756	2.669	2.989	2.907	4.696	4.643	7.883	7.610
$\beta=0.10$	6.375	6.449	6.234	5.901	6.234	5.901	6.364	5.980	7.883	7.610

Table 4 Dez reservoir capacities under different α and β and $\gamma=0.6$ for 480 month period

Model Function evaluation	$\alpha=0.70$		$\alpha=0.75$		$\alpha=0.80$		$\alpha=0.85$		$\alpha=0.9$	
	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP
$\beta=0.30$	3.111	2.99	3.205	2.962	4.617	4.472	6.685	6.451	9.508	9.156
$\beta=0.25$	3.111	2.99	3.205	2.962	4.617	4.472	6.707	6.451	9.508	9.156
$\beta=0.20$	3.111	2.99	3.205	2.962	4.617	4.472	6.707	6.451	9.508	9.156
$\beta=0.15$	3.186	3.05	3.205	2.962	4.617	4.472	6.707	6.451	9.508	9.156
$\beta=0.10$	6.125	6.01	6.125	6.01	6.125	6.01	6.707	6.451	9.508	9.156

significant. As an example, for $\gamma = 0.5$, and reliability levels of 0.70, 0.75, and 0.80, by decreasing β from 0.15 to 0.10 the rate of increase in reservoir capacity exceeds 100 % (Table 3).

It is observed that the design capacity will change as the reliability varies for a fixed values of β , and γ . For a fixed value of the reliability, however, the required design capacity may remain unchanged for some ranges of β and given γ . As an example, for $\alpha=0.70$ and $\gamma=0.6$ (Table 4), the required optimum reservoir capacity will remain 2.99 for values of β ranging from 0.30 to 0.20. It will increase to 3.05 if the value of β is decreased to 0.15. It is interesting to note how the required reservoir capacity will increase by almost 127 % (from 2.836 to 6.449 for $\alpha=0.70$) as the maximum permissible deficit in agricultural demand (β) is reduced from 0.15 to 0.10 (Table 3). As the required reliability increases, the reservoir capacity becomes less dependent on γ and the maximum permissible deficit in agricultural demand (β). Specifically speaking, for very high values of the reliability (i.e., $\alpha=0.9$), the reservoir capacity remains unchanged for all values of β (Table 4).

As γ increases to 0.8 (Table 6), the reservoir capacity becomes fully independent of the maximum permissible deficit in agricultural demand (β). In other words, for $\gamma=0.80$, the reservoir capacity is only a function of the required reliability. This is well illustrated in different columns of Table 6 where the optimum reservoir capacity is shown to be independent of the β .

In another word, as γ increases, sensitivity of the optimum reservoir capacity with β diminishes. Results presented in Table 6 clearly illustrate this fact. For $\gamma=0.8$, the optimum reservoir capacity is only dependent on the reliability level and remains constant with β ranging from 0.10 to 0.30. It is interesting to note that this observation held true when the GA-LP model was used. For $\gamma=0.7$, excluding the results for $\beta=0.1$ (with reliability levels of

Table 5 Dez reservoir capacities under different α and β and $\gamma=0.7$ for 480 month period

Model Function evaluation	$\alpha=0.70$		$\alpha=0.75$		$\alpha=0.80$		$\alpha=0.85$		$\alpha=0.9$	
	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP
$\beta=0.30$	5.046	4.587	5.963	5.668	7.310	7.039	9.031	8.548	12.236	11.134
$\beta=0.25$	5.046	4.587	5.963	5.668	7.310	7.039	9.031	8.548	12.236	11.134
$\beta=0.20$	5.046	4.587	5.963	5.668	7.310	7.039	9.031	8.548	12.236	11.134
$\beta=0.15$	5.046	4.587	5.963	5.668	7.310	7.039	9.031	8.548	12.236	11.134
$\beta=0.10$	6.455	6.042	6.455	6.042	7.310	7.039	9.031	8.548	12.236	11.134

Table 6 Dez reservoir capacities under different α and β and $\gamma=0.8$ for 480 month period

Model Function evaluation	$\alpha=0.70$		$\alpha=0.75$		$\alpha=0.80$		$\alpha=0.85$		$\alpha=0.9$	
	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP	GA-LP	ACO-LP
	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
$\beta=0.30$	9.559	8.935	9.687	9.069	10.343	9.825	14.343	12.661	17.792	15.656
$\beta=0.25$	9.559	8.935	9.687	9.069	10.343	9.825	14.343	12.661	17.792	15.656
$\beta=0.20$	9.559	8.935	9.687	9.069	10.343	9.825	14.343	12.661	17.792	15.656
$\beta=0.15$	9.559	8.935	9.687	9.069	10.343	9.825	14.343	12.661	17.799	15.656
$\beta=0.10$	9.559	8.935	9.687	9.069	10.343	9.825	14.343	12.661	17.799	15.656

0.70 and 0.75), the optimum reservoir capacity is dependent on the reliability levels and remains independent of β (Table 5).

In this problem and the way it was mathematically structured, for a reasonable fixed number of function evaluations, ACO-LP slightly outperforms GA-LP in locating the reliability optimum reservoir capacity for varying reliability and normalized deficit index. Although conflicting performances from different search-based algorithms have been reported, the extraction of solid conclusion had shown to be extremely difficult, if not impossible. Therefore, other search algorithms in the general hybrid structure could have equally been used to check the performance of the proposed ACO-LP algorithm. Please note that we do not seek to generalize our findings, make a statistical comparison of the results, and/or draw solid concluding remarks on their performances. Without trying to generalize our findings, the proposed ACO-LP algorithm slightly outperformed the GA-LP model for most of the cases tested in this simple example. Its similar performance in highly diverse water resources problems and their statistical analysis and comparison remains to be tested.

6 Conclusion

An ACO-LP hybrid model was proposed for optimum design of single water supply reservoir under reliability and volumetric deficit constraints. Efficient decomposition approach was developed and tested for the solution of the problem. The reliability-based optimization problem was formulated as a mixed integer program and solved with the proposed hybrid ACO-LP. The iterative penalty method easily and efficiently handled the reliability constraints in the solution procedure. The algorithm succeeded in solving the problem for all combinations of reliability levels and volumetric deficit index, as well as pre-specified fraction (γ) of the demand. It was shown how sensitivity of the optimum reservoir capacity with β diminished as γ increased. For the case under study, decreasing β from 0.85 to 0.90 for some small values of γ and reliability levels of 0.70, 0.75, and 0.80, doubled the required reservoir capacity. It was concluded that the sensitivity of the optimum reservoir capacity with β would diminish as γ increased.

It was observed that under special circumstances and under specific reliability values the common mixed integer solvers (i.e., branch-and-bound) may fail to identify the existing feasible solution for the problems with very narrow feasible zone. Under different reliability and volumetric deficit constraints and fixed number of function evaluations, Although operating rule was not included in the operational scheme, the procedure is capable of combining the operation rule with the design optimization.

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