

ESM 121

Water Science
and Management

Exercise 7:

Simulation Modeling

and

Optimization Modeling



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Simulation Modeling

The aim of this exercise section is to explore basic concepts of operating and allocation policies. Using the water available for each user, you will define equations that allocate water for the river and among 3 users.

River system

The water system is shown in Figure 1. There are 4 main water demands with different water requirements and priorities (Table 1).

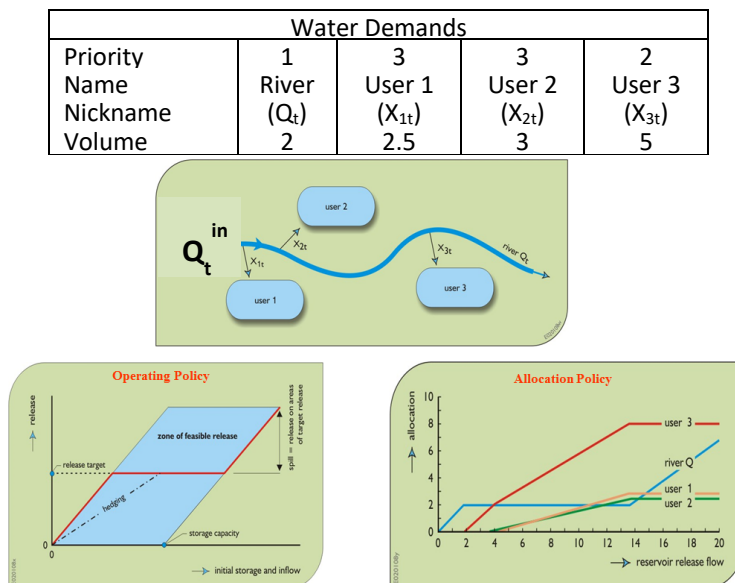


Figure 1

This is description of the water allocation policy for the system shown in Figure 1.

- *River* has the first priority, at least 2 units of water should be left in the river (if available) before the rest of the users can take their water demand. Once the water demand of all the user have been supplied, if there is any water available, then the remaining water will be left in the river.
- *User 3* has the second priority, and once the *River* has met its demand, then *User 3* will take as much water up to meet its water demand, which is 5 units.
- *User 1* and *User 2* share both the 3rd priority in the system. Once *River* and *User 3* have met their water demands, then *User 1* and *User 2* will take as much water as they can in

the same proportion up to their water demand is fulfilled, $User\ 1 = 2.5$ units of water and $User\ 2 = 3$ units of water.

To solve this exercise of water operation and allocation policy; first, you will estimate the allocation policy (Step 1), and second, you will *copy and paste* the formulas OR *lookup* this allocation policy when you are running the calculations (Step 2).

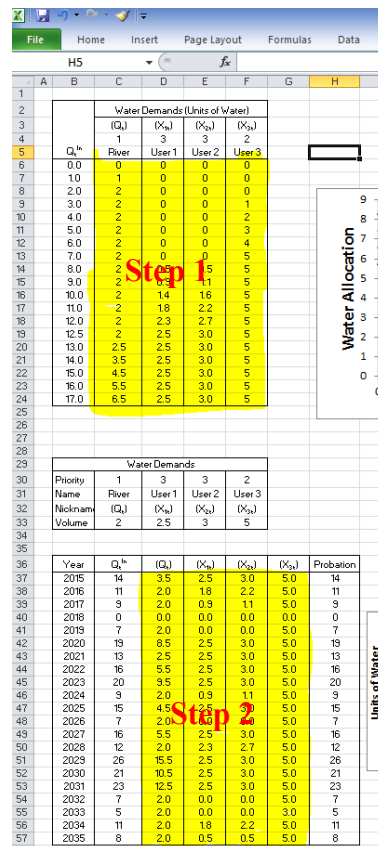


Figure 2. Step 1, water allocation policy and Step 2, mass balance model.

Defining the water allocation policies.

Water User: River

First, let's start working with *River*. According to the description provided by the allocation policy, water is left in the *River* using the following instructions:

“*River*” has the first priority, at least 2 units of water should be left in the river (if available) before the rest of the users can take their water demand. Once the water demand of all the user have been supplied, if there is any water available, then the remaining water will be left in the river.”

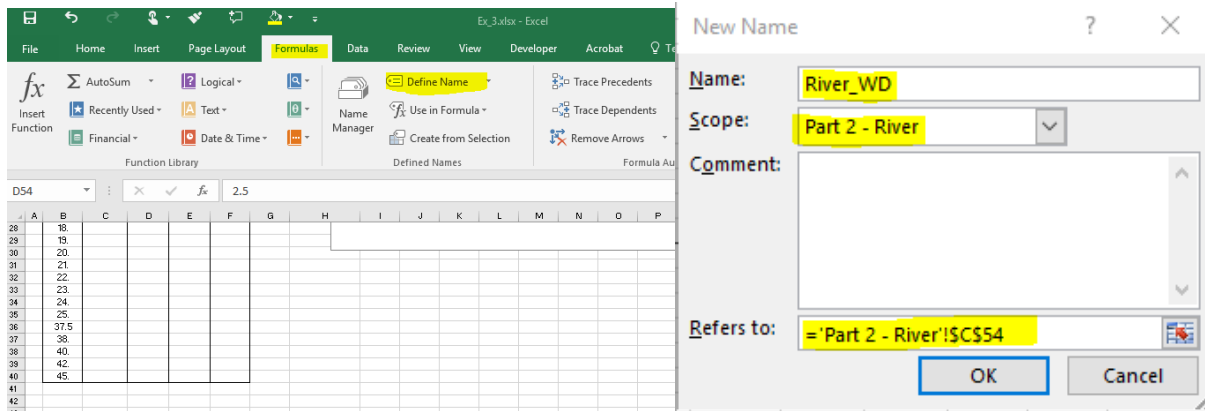
This can be converted into equations as follow:

$$Q_t^{River} = \begin{cases} Q_t^{in} & \text{if } Q_t^{in} < 2 \\ Q_t^{in} - \sum_{i=1}^{1=3} X_{i\ t} & \text{if } Q_t^{in} > 2 + \sum_{i=1}^{1=3} X_{i\ t} \\ 2 & \text{if } Q_t^{in} > 2 \text{ and } Q_t^{in} < 2 + \sum_{i=1}^{1=3} X_{i\ t} \end{cases}$$

The first condition, $Q_t^{River} = Q_t^{in}$, refers to “*River* [..], at least 2 units of water should be left in the river (if available)”, the second condition, $Q_t^{River} = 2$, refers to “*River* [..], at least 2 units of water should be left in the river (if available)” and the third condition, $Q_t^{River} = Q_t^{in} - \sum_{i=1}^{1=3} X_{i\ t}$, refers to “Once the water demand of all the user have been supplied [$2 + \sum_{i=1}^{1=3} X_{i\ t}$], if there is any water available, then the remaining water will be left in the river.”

Before declaring these equations, it will be easier if we define names to the variables first. Go to cell C54. Then, go to the menu of Formulas and click on “Define Name”. A dialogue window should come out, you should name the variable “River_WD” as a short name of “River Water Demand”. “. Very important, make sure that in “Scope:” drop down menu you select “**Part 2 - River**”. By default the “Refers to:” section should be selecting the cell C54 where the cursor is located. If this is not showing, click on the “cells and arrow” icon on the right and select the cell

C54. By doing this, you are naming that cell “River_WD” that you will be able to call it like that in excel, cool, isn’t it!



You will have to “Define Name” for the following variables and cells:

“Define Name”	Cell
Variable	
River_WD	C54
X1_WD	D54
X2_WD	E54
X3_WD	F54

If you had a problem naming your variables or not selecting the right cell, you can click on the Formulas/Name Manager where you can delete or edit you variables

The equation in Cell C6 is:

“=IF(B6<=River_WD,B6,IF(B6>River_WD+SUM(X1_WD,X2_WD,X3_WD),B6-SUM(X1_WD,X2_WD,X3_WD), River_WD))”

Ex_3.xlsx - Excel

File Home Insert Page Layout Formulas Data Review View Developer Acrobat Tell me what you want to do... Samuel Sa

Formulas

- AutoSum
- Logical
- Recently Used
- Text
- Financial
- Date & Time

Name Manager

- Define Name
- Use in Formula
- Create from Selection

Formula Auditing

- Trace Precedents
- Trace Dependents
- Remove Arrows
- Watch Window
- Calculation Options

Function Library

SUM = IF(B6<=River_WD,B6,IF(B6>River_WD+SUM(X1_WD,X2_WD,X3_WD),B6-SUM(X1_WD,X2_WD,X3_WD),River_WD))

A	B	C	D	E	F	G	H	I	J	K
1										
2		Water Allocation (Units of Water)								
3		(Q _t ^{River})	(X _{1t})	(X _{2t})	(X _{3t})					
4		1	3	3	2					
5		River	User 1	User 2	User 3					
6	Qt ^{In}	=IF(B6<=Riv								
7	1.	1.								

9. —(QtRiver

8.

Priority	1	3	3	2
Name	River	User 1	User 2	User 3
Nickname	(Q _t ^{River})	(X _{1t})	(X _{2t})	(X _{3t})
Volume	2	2.5	3	5

Figure 3

The first part of this equation, grey box, refers to the first condition, the green box refers to the third condition, and if none of the previous conditions applied, then it applies the result from condition 2.

$$=IF(B6 \leq \text{River_WD}, B6, IF(B6 > \text{River_WD} + \text{SUM}(X1_WD, X2_WD, X3_WD), B6 - \text{SUM}(X1_WD, X2_WD, X3_WD), \text{River_WD}))$$

$$River_t = \begin{cases} Q_t^{in} & \text{if } Q_t^{in} < 2 \\ 2 & \text{if } Q_t^{in} > 2 \text{ and } Q_t^{in} < 2 + \sum_{i=1}^3 X_{i,t} \\ Q_t^{in} - \sum_{i=1}^3 X_{i,t} & \text{if } Q_t^{in} > 2 + \sum_{i=1}^3 X_{i,t} \end{cases}$$

Figure 4

Copy and paste this equation for the remaining of the column.

G24					
	A	B	C	D	E
1					
2			Water Demands (Units of Water)		
3			(Q _t ^{River})	(X _{1t})	(X _{2t})
4			1	3	3
5		Q _t ⁱⁿ	River	User 1	User 2
6		0.	0		
7		1.	1		
8		2.	2		
9		3.	2		
10		4.	2		
11		4.5	2		
12		5.	2		
13		5.5	2		
14		6.	2		
15		6.5	2		
16		7.	2		
17		8.	2		
18		9.	2		
19		10.	2		
20		11.	2		
21		12.	2		
22		12.5	2		
23		13.	2.5		
24		14.	3.5		
25		15.	4.5		
26		16.	5.5		
27		17.	6.5		
28		18.	7.5		
29		19.	8.5		
30		20.	9.5		
31		21.	10.5		
32		22.	11.5		
33		23.	12.5		
34		24.	13.5		
35		25.	14.5		
36		37.5	27		
37		38.	27.5		
38		40.	29.5		
39		42.	31.5		
40		45.	34.5		
41					

Figure 5

Water User: User 3

According to the description provided by the allocation policy, *User 3* has the second priority, and once the *River* has met its demand, then *User 3* will take as much water up to its water demand is fulfilled, which is 5 units.

This can be converted into equations as follow:

$$X_{3t} = \begin{cases} 0 & \text{if } Q_t^{in} \leq 2 \\ Q_t^{in} - Q_t^{River} & \text{if } Q_t^{in} > Q_t^{River} \text{ and } Q_t^{in} < Q_t^{River} + X_{3t} \\ 5 & \text{if } Q_t^{in} \geq Q_t^{River} + X_{3t} \end{cases}$$

The first condition, $X_{3t}=0$, refers to “*User 3* has the second priority, and once the *River* has met its demand, then *User 3* [...]” the second condition, $X_{3t}=5$, refers to “*User 3* [...] will take as much water up to meet its water demand, which is 5 units.” and the third condition, $X_{3t}=Q_t^{in}-Q_t^{River}$ refers to “*User 3* [...] will take as much water up to its water demand is fulfilled, which is 5 units.”

The equation in Cell F6 is:

“=IF(B6<=River_WD,0,IF(AND(B6>River_WD,B6<River_WD+X3_WD),B6-River_WD,X3_WD))”

SUM									
	A	B	C	D	E	F	G	H	I
1									
2			Water Allocation (Units of Water)						
3			(Q _t ^{River})	(X _{1t})	(X _{2t})	(X _{3t})			
4			1	3	3	2			
5		Q _t ⁱⁿ	River	User 1	User 2	User 3			9.
6		0.	0.			3_WD))			8.
7		1.	1.			0.			
49									
50			Water Demands						
51		Priority	1	3	3	2			
52		Name	River	User 1	User 2	User 3			
53		Nickname	(Q _t ^{River})	(X _{1t})	(X _{2t})	(X _{3t})			
54		Volume	2	2.5	3	5			
55									

Figure 6

The first part of this equation, grey box, refers to the first condition, the green box refers to the third condition, and if none of the previous conditions applied, then it applies the result from condition

2.

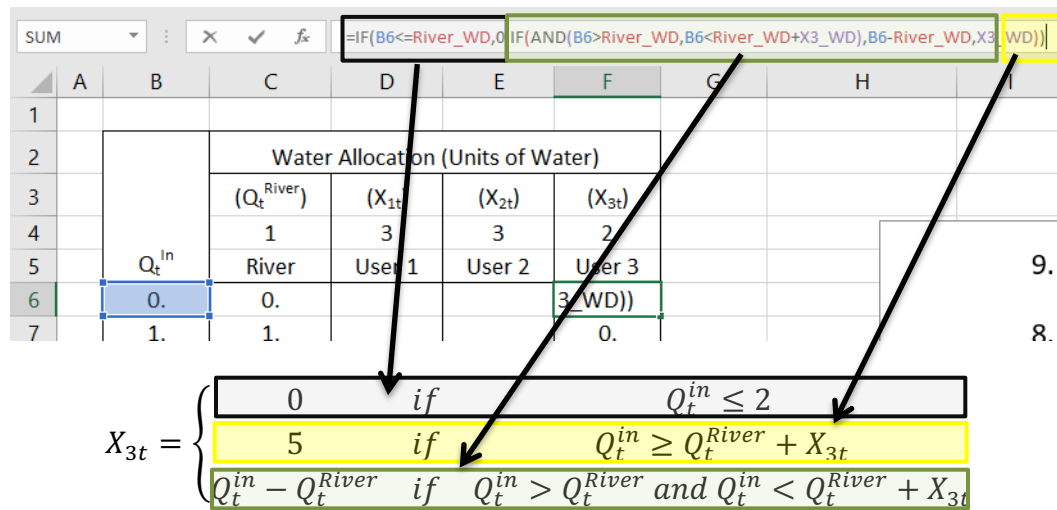


Figure 7

Copy and paste this equation for the remaining of the column.

	A	B	C	D	E	F
1						
2			Water Demands (Units of Water)			
3			(Q _t ^{River})	(X _{1t})	(X _{2t})	(X _{3t})
4			1	3	3	2
5		Q _t ^{In}	River	User 1	User 2	User 3
6		0.	0			0
7		1.	1			0
8		2.	2			0
9		3.	2			1
10		4.	2			2
11		4.5	2			2.5
12		5.	2			3
13		5.5	2			3.5
14		6.	2			4
15		6.5	2			4.5
16		7.	2			5
17		8.	2			5
18		9.	2			5
19		10.	2			5
20		11.	2			5
21		12.	2			5
22		12.5	2			5
23		13.	2.5			5
24		14.	3.5			5
25		15.	4.5			5
26		16.	5.5			5
27		17.	6.5			5
28		18.	7.5			5
29		19.	8.5			5
30		20.	9.5			5
31		21.	10.5			5
32		22.	11.5			5
33		23.	12.5			5
34		24.	13.5			5
35		25.	14.5			5
36		37.5	27			5
37		38.	27.5			5
38		40.	29.5			5
39		42.	31.5			5
40		45.	34.5			5
41						
42						

Figure 8

Water User: User 1 and 2

According to the description provided by the allocation policy, *User 1* and *User 2* share both the 3rd priority in the system. Once *River* and *User 3* have met their water demands, then *User 1* and *User 2* will take as much water as they can in the same proportion up to meet their water demand, $User\ 1 = 2.5$ units of water and $User\ 2 = 3$ units of water.

For *User 1*, this can be converted into equations as follow:

$$X_{1t} = \begin{cases} 0 & \text{if } Q_t^{\text{in}} \leq Q_t^{\text{River}} + X_{3t} \\ \left(Q_t^{\text{in}} - (Q_t^{\text{River}} + X_{3t}) \right) \times \left(\frac{X_{2t}}{X_{1t} + X_{2t}} \right) & \text{if } Q_t^{\text{in}} - (Q_t^{\text{River}} + X_{3t}) < X_{1t} + X_{2t} \text{ and } Q_t^{\text{in}} > Q_t^{\text{River}} + X_{3t} \\ 2.5 & \text{if } Q_t^{\text{in}} - (Q_t^{\text{River}} + X_{3t}) > X_{1t} + X_{2t} \end{cases}$$

For User2, this description can be converted unto equations as follows:

$$X_{2t} = \begin{cases} 0 & \text{if } Q_t^{\text{in}} \leq Q_t^{\text{River}} + X_{3t} \\ \left(Q_t^{\text{in}} - (Q_t^{\text{River}} + X_{3t}) \right) \times \left(\frac{X_{1t}}{X_{1t} + X_{2t}} \right) & \text{if } Q_t^{\text{in}} - (Q_t^{\text{River}} + X_{3t}) < X_{1t} + X_{2t} \text{ and } Q_t^{\text{in}} > Q_t^{\text{River}} + X_{3t} \\ 3 & \text{if } Q_t^{\text{in}} - (Q_t^{\text{River}} + X_{3t}) > X_{1t} + X_{2t} \end{cases}$$

The first condition, $X_{1t}=0$, refers to “Once River and User 3 have met their water demands, then User 1 and User 2 will take [...],” the second condition, $X_{1t}=2.5$, refers to “[...] User 1 and User 2 will take as much water as they can in the same proportion up to meet their water demand, User 1 = 2.5 units of water and User 2 = 3 units of water” and the third condition, refers to “[...] User 1 and User 2 will take as much water as they can in the same proportion up to meet their water demand, User 1 = 2.5 units of water and User 2 = 3 units of water”

The equation for *User 1* in Cell D6 is: “=IF(B6<=River_WD+X3_WD,0,IF(B6-(River_WD+X3_WD)>=X1_WD+X2_WD,X1_WD,(B6-(River_WD+X3_WD))*(X1_WD/(X1_WD+X2_WD))))”

Excel formula bar: =IF(B6<=River_WD+X3_WD,0,IF(B6-(River_WD+X3_WD)>=X1_WD+X2_WD,X1_WD,(B6-(River_WD+X3_WD))*(X1_WD/(X1_WD+X2_WD))))

1					
2		Water Allocation (Units of Water)			
3		(Q _t ^{River})	(X _{1t})	(X _{2t})	(X _{3t})
4		1	3	3	2
5	Q _t ⁱⁿ	River	User 1	User 2	User 3
6	0.	0.	_WD)))))		0.
7	1.	1.	0.		0.
8	2.	2.	0.		0.
9	3.	2.	0.		1.

49					
50		Water Demands			
51	Priority	1	3	3	2
52	Name	River	User 1	User 2	User 3
53	Nickname	(Q_t^{River})	(X_{1t})	(X_{2t})	(X_{3t})
54	Volume	2	2.5	3	5
55					

Figure 9

The first part of this equation, grey box, refers to the first condition, the green box refers to the third condition, and if none of the previous conditions applied, then it applies the result from condition

2.

$$X_{1t} = \begin{cases} 0 & \text{if } Q_t^{in} \leq Q_t^{River} + X_{3t} \\ 2.5 & \text{if } Q_t^{in} - (Q_t^{River} + X_{3t}) > X_{1t} + X_{2t} \\ \left(Q_t^{in} - (Q_t^{River} + X_{3t}) \right) \times \left(\frac{X_{2t}}{X_{1t} + X_{2t}} \right) & \text{if } Q_t^{in} - (Q_t^{River} + X_{3t}) < X_{1t} + X_{2t} \text{ and } Q_t^{in} > Q_t^{River} + X_{3t} \end{cases}$$

The equation for *User 2* is very similar to *User 1*, in Cell E6 is:

“=IF(B6<=River_WD+X3_WD,0,IF(B6-(River_WD+X3_WD)>=X1_WD+X2_WD,X2_WD,(B6-(River_WD+X3_WD))*(X2_WD/(X1_WD+X2_WD))))”

Copy and paste this equation for the remaining of the column.

	A	B	C	D	E	F
1						
2			Water Demands (Units of Water)			
3			(Q_t^{River})	(X_{1t})	(X_{2t})	(X_{3t})
4			1	3	3	2
5		Q_t^{In}	River	User 1	User 2	User 3
6		0.	0.	0.	0.	0.
7		1.	1.	0.	0.	0.
8		2.	2.	0.	0.	0.
9		3.	2.	0.	0.	1.
10		4.	2.	0.	0.	2.
11		4.5	2.	0.	0.	2.5
12		5.	2.	0.	0.	3.
13		5.5	2.	0.	0.	3.5
14		6.	2.	0.	0.	4.
15		6.5	2.	0.	0.	4.5
16		7.	2.	0.	0.	5.
17		8.	2.	0.5	0.5	5.
18		9.	2.	0.9	1.1	5.
19		10.	2.	1.4	1.6	5.
20		11.	2.	1.8	2.2	5.
21		12.	2.	2.3	2.7	5.
22		12.5	2.	2.5	3.	5.
23		13.	2.5	2.5	3.	5.
24		14.	3.5	2.5	3.	5.
25		15.	4.5	2.5	3.	5.
26		16.	5.5	2.5	3.	5.
27		17.	6.5	2.5	3.	5.
28		18.	7.5	2.5	3.	5.
29		19.	8.5	2.5	3.	5.
30		20.	9.5	2.5	3.	5.
31		21.	10.5	2.5	3.	5.
32		22.	11.5	2.5	3.	5.
33		23.	12.5	2.5	3.	5.
34		24.	13.5	2.5	3.	5.
35		25.	14.5	2.5	3.	5.
36		37.5	27.	2.5	3.	5.
37		38.	27.5	2.5	3.	5.
38		40.	29.5	2.5	3.	5.
39		42.	31.5	2.5	3.	5.
40		45.	34.5	2.5	3.	5.
41						

Figure 10

The graph should look like:

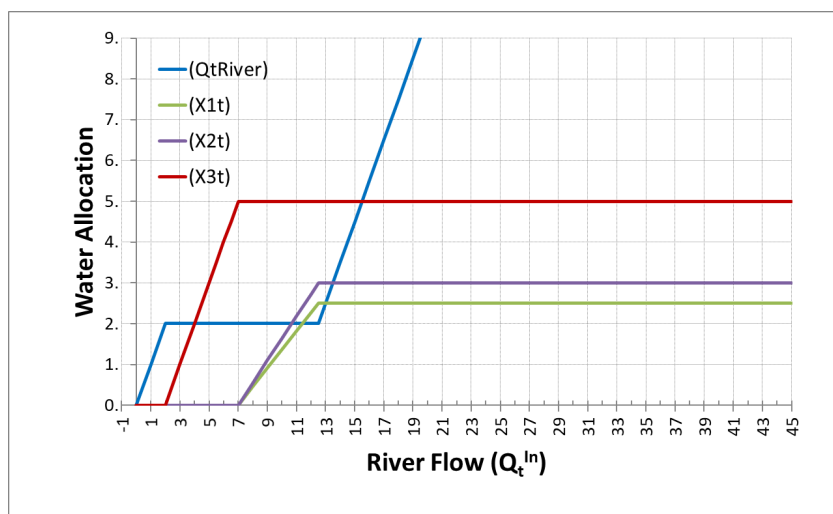


Figure 11

Linking the Water Allocation policy with the Mass Balance Model

There are two methods to use the allocation policy that was just derived into the *mass balance model*. Use **EITHER** of the following two methods, **but only chose one**, the one that you feel more comfortable using.

First Method

The easiest way is to select cells C6 to F6 (C6:F6), copy (Ctrl+C) and paste the formulas on cells into cells D45 to G45 (D45:G45).

						D45		fx		=IF(C45<=\$C\$41,C45,IF(C45>\$C\$41+SUM(\$D\$41:\$				
1	A	B	C	D	E	F								
2		Q _t ^{In}	Water Demands (Units of Water)				A	B	C	D	E	F	G	H
3			(Q _t ^{River})	(X _{1t})	(X _{2t})	(X _{3t})	40	Nickname	(Q _t ^{River})	(X _{1t})	(X _{2t})	(X _{3t})		
4			1	3	3	2	41	Volume	2	2.5	3	5		
5			River	User 1	User 2	User 3	42							
6			0.	0	0	0	0	43	Mass Balance Model					
7		1.	1	0	0	0	44	Year	Q _t ^{In}	(Q _t ^{River})	(X _{1t})	(X _{2t})	(X _{3t})	Probation
							45	2015	14	3.5	2.5	3	5	14
							46	2016	11					0

Figure 12. On the left, Copy (Ctrl+C) cells C6 to F6, and paste them (Ctrl+V) into cells D45 to G45

And then use these cells (D45:G45) to copy then through the rest of the table.

E48		fx		=IF(C48<=\$C\$41+\$F\$41,0,IF(C48-(\$C\$41+\$F\$41):				
A	B	C	D	E	F	G	H	
41	Volume	2	2.5	3	5			
42								
43	Mass Balance Model							
44	Year	Q_t^{in}	(Q_t^{river})	(X_{1t})	(X_{2t})	(X_{3t})	Probation	
45	2015	14	3.5	2.5	3.	5	14	
46	2016	11	2	1.8	2.2	5	11	
47	2017	9	2	0.9	1.1	5	9	
48	2018	0	0	0.	0.	0	0	
49	2019	7	2	0.	0.	5	7	
50	2020	19	8.5	2.5	3.	5	19	
51	2021	13	2.5	2.5	3.	5	13	
52	2022	16	5.5	2.5	3.	5	16	
53	2023	20	9.5	2.5	3.	5	20	
54	2024	9	2	0.9	1.1	5	9	
55	2025	15	4.5	2.5	3.	5	15	
56	2026	7	2	0.	0.	5	7	
57	2027	16	5.5	2.5	3.	5	16	
58	2028	12	2	2.3	2.7	5	12	
59	2029	26	15.5	2.5	3.	5	26	
60	2030	21	10.5	2.5	3.	5	21	
61	2031	23	12.5	2.5	3.	5	23	
62	2032	7	2	0.	0.	5	7	
63	2033	5	2	0.	0.	3	5	
64	2034	11	2	1.8	2.2	5	11	
65	2035	8	2	0.5	0.5	5	8	
66								

Figure 13

Second Method

The second way to do this, is a little bit more elegant, but still the same principle. This method is used, and it is very helpful, when managing large amount of water users. In this case, what you want is to have control of the water allocating policy in one table (B6:F40) and to recall values from this table in the mass balance model (B59:H79). You can create a “lookup” command that will find the values estimated in the allocation policy table. In cell D59, let’s insert a command that will look up for the value of Q_t^{River} in the table water allocation table using the following equation:

“=VLOOKUP(\$C59,\$B\$6:\$F\$40,2,TRUE)”

What the previous command means, is that excel will lookup vertically (“VLookup”) for the Q_t^{in} value of cell C59 (in this case 14) in the first column (it always start looking in the first column) of the table specified by the cell coordinates “\$B\$6:\$F\$40” (the green box in the following figure),

and it will retrieve the value of column number 2 of the specified table, which is Q_t^{River} , in this case for 14 is 3.5. When using this command, make sure to specify the column number that you want values to retrieve, and excel will always use the first column as the column to lookup for numbers, in this case, it will look for the “approximate” value in cell C58 on the first column of the table specified, that’s why at the end of the command you need to write “TRUE”.

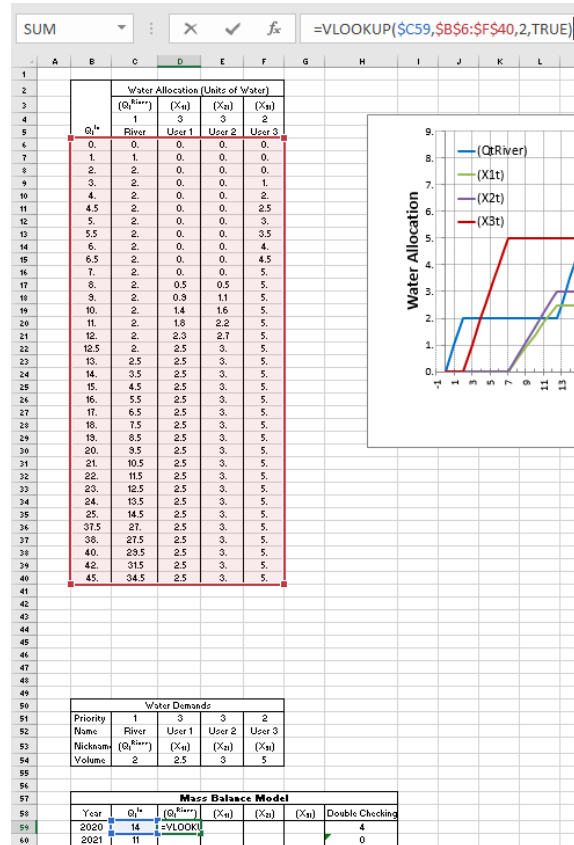


Figure 14

For *User 1*, X_{1t} , use the following equation in cell E59:

“=VLOOKUP(\$C59,\$B\$6:\$F\$40,3,TRUE)”

For *User 2*, X_{2t} , use the following equation in cell F59:

“=VLOOKUP(\$C59,\$B\$6:\$F\$40,4,TRUE)”

For *User 3*, X_{3t} , use the following equation in cell G59:

“=VLOOKUP(\$C59,\$B\$6:\$F\$40,5,TRUE)”

Results from the Mass Balance model should look like the following pictures

E48		fx		=IF(C48<=\$C\$41+\$F\$41,0,IF(C48-(\$C\$41+\$F\$41):			
A	B	C	D	E	F	G	H
41	Volume	2	2.5	3	5		
42							
43	Mass Balance Model						
44	Year	Q_t^{in}	(Q_t^{River})	(X_{1t})	(X_{2t})	(X_{3t})	Probation
45	2015	14	3.5	2.5	3.	5	14
46	2016	11	2	1.8	2.2	5	11
47	2017	9	2	0.9	1.1	5	9
48	2018	0	0	0.	0.	0	0
49	2019	7	2	0.	0.	5	7
50	2020	19	8.5	2.5	3.	5	19
51	2021	13	2.5	2.5	3.	5	13
52	2022	16	5.5	2.5	3.	5	16
53	2023	20	9.5	2.5	3.	5	20
54	2024	9	2	0.9	1.1	5	9
55	2025	15	4.5	2.5	3.	5	15
56	2026	7	2	0.	0.	5	7
57	2027	16	5.5	2.5	3.	5	16
58	2028	12	2	2.3	2.7	5	12
59	2029	26	15.5	2.5	3.	5	26
60	2030	21	10.5	2.5	3.	5	21
61	2031	23	12.5	2.5	3.	5	23
62	2032	7	2	0.	0.	5	7
63	2033	5	2	0.	0.	3	5
64	2034	11	2	1.8	2.2	5	11
65	2035	8	2	0.5	0.5	5	8
66							

Figure 15

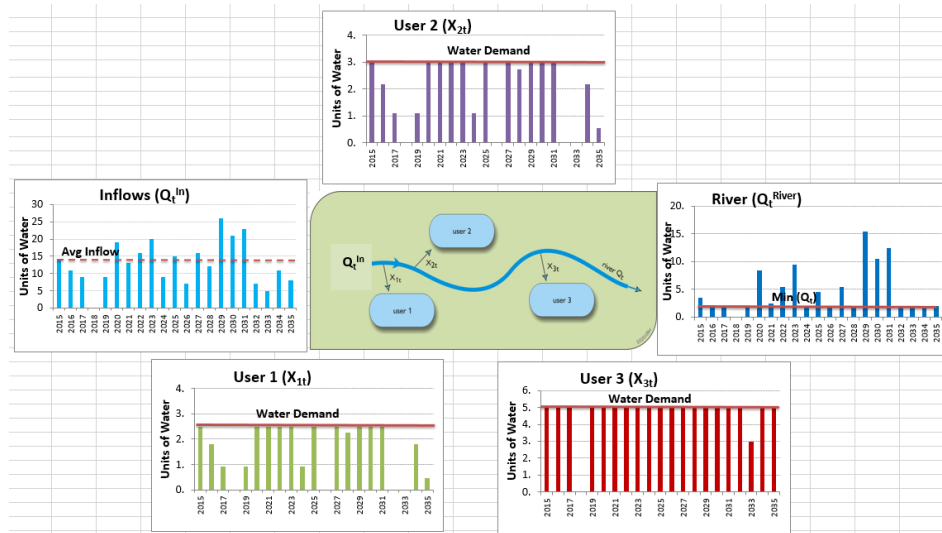


Figure 16

To be turned in:

- 1) Show a table like the one calculated in Figure 15 and the series of graphs calculated in Figure 16.

Water Resources Performance Criteria

Phew, we have done a lot of work to describe the water allocation system for four water demands (River, X1, X2, and X3), imagine in a basin where we have hundreds and sometimes thousands of users! All the previous work quantitatively describes how water is allocated, but it does not give us any performance of the water supply. For estimating the water supply performance, there are four water resources performance criteria used widely in the field: Reliability (in time and volume), Vulnerability, and Resilience. I will show you how to calculate those water resources performance criteria to evaluate the our water resources system.

Reliability (Time and Volume)

Have you ever heard in the news about “Water Supply Reliability”? do you remember Governor Brown in our last drought when he talked about “making sure everyone had a reliable water supply during the drought? Water Supply Reliability is a common term used to quantify how frequent a water demand is met (time-based reliability), and how close to the total amount demanded (volumetric reliability). The *time based reliability* is the portion of time that water demand is fully supplied, or in other words, how frequent a water demand is fully supplied.

To estimate the time based reliability, we need to estimate *water supply deficits*, which is the difference between the water demand minus the water supplied:

$$Deficit_t^i = \begin{cases} X_{Demand,t}^i - X_{Supplied,t}^i & \text{if } X_{Demand,t}^i > X_{Supplied,t}^i \\ 0 & \text{if } X_{Demand,t}^i = X_{Supplied,t}^i \end{cases}$$

where: $X_{Demand,t}^i$ is the water demand for the i^{th} water user, and $X_{Supplied,t}^i$ is the water supplied in the t^{th} time period. Finally, the reliability for the i^{th} user is:

$$Reliability(time)^i = \frac{\# \text{ of times } Deficit_t^i = 0}{n}$$

where n is the total number of time steps, often used the total number of years or months, depending on the water management unit of time.

The volumetric reliability is the sum of the deficits divided by sum of the water demand. The volumetric reliability express how much water was supplied compared with the total amount of water demanded.

$$Reliability(volume)^i = \frac{\sum X_{Supplied,t}^i}{\sum X_{Demand,t}^i}$$

First, let's calculate the matrix of Water Supply Deficits. As mentioned before, a water supply deficit is the difference between the water demand minus the water supplied. For the River demand, a deficit only exist if the water supplied is less than the water demand ($Q_t^{River} = 2$ units of water). Let's estimate the deficit for year 2020 in cell D87 as follows:

“=IF(D59<River_WD,River_WD-D59,0)”

of deficits -> Cell D110: “=COUNTIF(D87:D107,”>0”)”

of NO deficits -> Cell D111: “=COUNTIF(D87:D107,”=0”)”

Sum of deficits -> Cell D112: “=SUMIF(D87:D107,”>0”)”

Sum of Water Demand -> Cell D113: “=River_WD*D109”

Sum of Water Supplied -> Cell D114: “=D113-D112”

Water Supply Deficits					
Year	(Q _r ^{river})	(X ₂₁)	(X ₂₂)	(X ₂₃)	
2020	0				
2021	0				
2022	0				
2023	2				
2024	0				
2025	0				
2026	0				
2027	0				
2028	0				
2029	0				
2030	0				
2031	0				
2032	0				
2033	0				
2034	0				
2035	0				
2036	0				
2037	0				
2038	0				
2039	0				
2040	0				
n =		21			
# of deficits =		1			
# of NO deficits =		20			
Sum of deficits =		2			
Sum of the Water Demand =		42			
Sum of the Water Supplied =		40			

According to the equation of reliability in time, we can estimate it as follows:

$$Reliability(time)^i = \frac{\# \text{ of times } Deficit_t^i = 0}{n}$$

Reliability in time -> Cell D116: “=D111/D109”. The time-based reliability means that 95% of the time the water demand of the river can be fully supplied.

SUM				=D111/D109
	A	B	C	D
108				
109			n =	21
110			# of deficits =	1
111			# of NO deficits =	20
112			Sum of deficits =	2
113			Sum of the Water Demand =	42
114			Sum of the Water Supplied =	40
115				
116			Reliability (time) =	=D111/D109
117			Reliability (volume) =	95%
118			Vulnerability =	
119			Resilience =	

According to the equation of volumetric reliability, we can estimate it as follows:

$$Reliability(volume)^i = \frac{\sum X_{Supplied,t}^i}{\sum X_{Demand,t}^i}$$

Volumetric Reliability -> Cell D117: “=D114/D113”. The volumetric reliability means that 95% of the volume requested by the River was able to be supplied.

SUM				=D114/D113
	A	B	C	D
108				
109			n =	21
110			# of deficits =	1
111			# of NO deficits =	20
112			Sum of deficits =	2
113			Sum of the Water Demand =	42
114			Sum of the Water Supplied =	40
115				
116			Reliability (time) =	95%
117			Reliability (volume) =	4/D113
118			Vulnerability =	
119			Resilience =	

Vulnerability

The performance criterion of vulnerability expresses the severity of the deficits. In summary, it is the average of the water supply deficits, expressed as a percentage of the water demand.

Vulnerability can be calculated as follows:

$$Vulnerability^i = \frac{\left(\frac{\sum Deficit_t^i}{\# \text{ of times } Deficit_t^i > 0 \text{ occurred}} \right)}{Water Demand^i}$$

Vulnerability -> Cell D118: “=(D112/D110)/River_WD”. The vulnerability criterion means that when a deficit happens, on average, the magnitude of the deficit is 100% of the water demanded by the river.

SUM						
	A	B	C	D	E	F
108						
109			n =	21		
110			# of deficits =	1		
111			# of NO deficits =	20		
112			Sum of deficits =	2		
113			Sum of the Water Demand =	42		
114			Sum of the Water Supplied =	40		
115						
116			Reliability (time) =	95%		
117			Reliability (volume) =	95%		
118			Vulnerability =	=(D112/D110)/River_WD		
119			Resilience =			
120						
121						

One tricky thing about the Vulnerability is to calculate it when no deficit occur. For this case, the system is considered 0% vulnerable. Thus, the equation in Cell D118 should be changed to account for this peculiar condition as follows: “=IF(D110=0,0,(D112/D110)/River_WD)”.

SUM						
	A	B	C	D	E	F
108						
109			n =	21		
110			# of deficits =	1		
111			# of NO deficits =	20		
112			Sum of deficits =	2		
113			Sum of the Water Demand =	42		
114			Sum of the Water Supplied =	40		
115						
116			Reliability (time) =	95%		
117			Reliability (volume) =	95%		
118			Vulnerability =	IF(D110=0,0,(D112/D110)/River_WD)		
119			Resilience =	100%		
120			Sustainability Index =	98%		
121						

shows that only in year 2024 the previous condition is true, meaning that the previous year (2023) was in a water supply deficit and that the current year (2024) it is not.

Year	Water Supply Deficits					Resilience			
	(Q _k ^{River})	(X _{1k})	(X _{2k})	(X _{3k})		(Q _k ^{River})	(X _{1k})	(X _{2k})	(X _{3k})
2020	0					0	0	0	0
2021	0					0			
2022	0					0			
2023	2					0			
2024	0					1			
2025	0					0			
2026	0					0			
2027	0					0			
2028	0					0			
2029	0					0			
2030	0					0			
2031	0					0			
2032	0					0			
2033	0					0			
2034	0					0			
2035	0					0			
2036	0					0			
2037	0					0			
2038	0					0			
2039	0					0			
2040	0					0			

You can sum all of the times this condition occurs in Cell I109: “=SUM(I87:I107)”

The resilience can be calculated as follows:

$$Res^i = \frac{\# \text{ of times } Deficit_t^i = 0 \text{ follows } Deficit_t^i > 0}{\# \text{ of times } Deficit_t^i > 0 \text{ occurred}}$$

Resilience -> Cell D119: “=IF(I109=0,1,I109/D110)”. The resilience criterion means the probability that a water demands recovers from a deficit, when they occur. For the River water demand, there is a 100% probability that the system will recover.

Water Supply Deficits					Resilience			
Year	(Q _d - Q _{river})	(X _{1t})	(X _{2t})	(X _{3t})	(Q _d - Q _{river})	(X _{1t})	(X _{2t})	(X _{3t})
2020	0				0	0	0	0
2021	0				0			
2022	0				0			
2023	2				0			
2024	0				1			
2025	0				0			
2026	0				0			
2027	0				0			
2028	0				0			
2029	0				0			
2030	0				0			
2031	0				0			
2032	0				0			
2033	0				0			
2034	0				0			
2035	0				0			
2036	0				0			
2037	0				0			
2038	0				0			
2039	0				0			
2040	0				0			
n =	21				Count Resilience =	1		
# of deficits =	1							
# of NO deficits =	20							
Sum of deficits =	2							
Sum of the Water Demand =	42							
Sum of the Water Supplied =	40							
Reliability (time) =	95%							
Reliability (volume) =	95%							
Vulnerability =	100%							
Resilience =	100%							

One tricky thing about the Resilience is to calculate the resilience of a system when no deficit occurs. For this case, the system is considered 100% resilient. Thus, the equation in Cell D119 should be changed to account for this peculiar condition as follows: “=IF(I109=0,1,I109/D110)”.

SUM									
	A	B	C	D	E	F	G	H	I
108			n =	21				Count Resilience =	1
109			# of deficits =	1					
110			# of NO deficits =	20					
111			Sum of deficits =	2					
112			Sum of the Water Demand =	42					
113			Sum of the Water Supplied =	40					
114									
115			Reliability (time) =	95%					
116			Reliability (volume) =	95%					
117			Vulnerability =	100%					
118			Resilience =	=IF(I109=0,1,I109/D110)					
119			Sustainability Index =						
120									
121									

Water Resources Sustainability Index

Now, think about this: How do you know that a given user is receiving adequately their water demand? So far, we have estimated four performance criteria, but imagine that we start evaluating several alternative water management strategies and all of a sudden some criteria starts going up other going down. Can we estimate a single number (an Index) that summarize the performance

of a given water demand? In order to summarize all the performance criteria, we used index that helps synthesizing results. The Water Resources Sustainability Index (SI) is an index that aggregates a set of performance criteria, it is the geometric average of a group of m performance criteria for a given water user i :

$$SI^i = \left[\prod_{m=1}^M \text{Performance Criteria}_m^i \right]^{1/M}$$

For our case, the SI can be expressed as:

$$SI^i = [Rel(time)^i * Rel(Volume)^i * Res^i * (1 - Vuln^i)]^{1/4}$$

For the River water demand, the SI can be estimated in Cell D120 as follows:

Cell D120: “=(D116*D117*D119*(1-D118))^(1/4)”

SUM								
	A	B	C	D	E	F	G	H
108								
109			n =	21				Count Resilience=
110			# of deficits =	1				
111			# of NO deficits =	20				
112			Sum of deficits =	2				
113			Sum of the Water Demand =	42				
114			Sum of the Water Supplied =	40				
115								
116			Reliability (time) =	95%				
117			Reliability (volume) =	95%				
118			Vulnerability =	100%				
119			Resilience =	100%				
120			Sustainability Index =	=(D116*D117*D119*(1-D118))^(1/4)				
121								

Figure 17. Results of the Performance Criteria and Sustainability Index for the River Water demand

For the River water demand, the SI = 0% meaning that is unsustainable. While the Reliability (in time and volume) is very high (95%) and the resilience is very high (100%), when a deficit occurs, this deficit is very damaging, 100% of the water demand. That's the reason why the SI = 0%

Instructions

Calculate the deficit matrix for users 1, 2 and 3.

Hint: for year 2020, the deficit can be calculated as follows:

User1 -> Cell E87: “=IF(E59<X1_WD,**X1_WD**-E59,0)”

User2 -> Cell F87: “=IF(F59<X2_WD,**X2_WD**-F59,0)”

User3 -> Cell G87: “=IF(G59<X3_WD,**X3_WD**-G59,0)”

Calculate the following variables for Users 1, 2 and 3: n, # of deficits, # of NO deficits, Sum of deficits, Sum of Water Demand, and Sum of Water Supplied

Hint: Remember to use the appropriate water demands for user 1 (**X1_WD**), 2 (**X2_WD**), and 3 (**X3_WD**) in the equation of Sum of Water Demand.

Calculate the resilience matrix for Users 1, 2 and 3

Estimate the Reliability (time-based and volumetric), Vulnerability, resilience and Water resources Sustainability Index for user 1, 2 and 3.

Hint: Remember to use the appropriate water demands for user 1 (**X1_WD**), 2 (**X2_WD**), and 3 (**X3_WD**) in the equation of Vulnerability.

To be turned in:

- 1) Show table (like figure 17) of the four performance criteria [Reliability(time and volume), vulnerability and resilience) and the Sustainability Index.

- 2) Change the value of the *River* water demand (Q_t^{River}) to 4 units in cell C54. Show a table like the one calculated in Figure 15 and the series of graphs calculated in Figure 16.

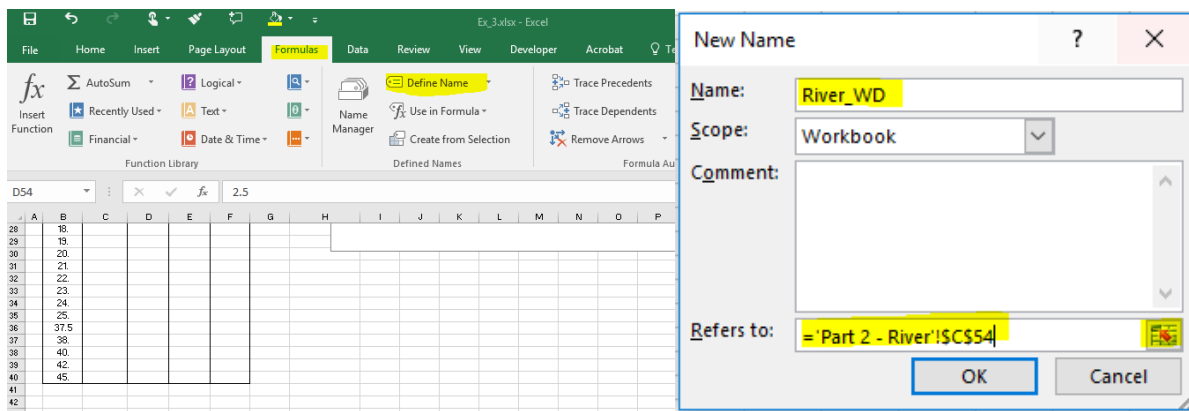
- 3) Show table (like figure 17) of the four performance criteria [Reliability(time and volume), vulnerability and resilience) and the Sustainability Index for all water demands for $Q_t^{River} = 4$ units. Does the water supply for *Users 1, 2 and 3* change? Does it improved or got worse?

- 4) Calculate the average annual flow coming from upstream (Q_t^{In}) (Average of cells C59:C79). How much the River water demand represent in comparison to the average annual inflows [$River_WD / Avg. (Q_t^{In})$]? Do you think this is enough water so the ecosystem can be sustained? Imagine that from your total monthly income (let say \$2,000), this monthly stipend is reduced to the same percentage of water that was left in the river (let say 30% of \$2,000 = \$600). Can you adequately live with this budget in a month? Do you see any similitude with what we have left to the river? Elaborate why there may be a conflict with human and environmental water supply.

River and Reservoir System

Now, you are going to be working with a reservoir system, how exciting!

Before declaring any equation, it will be easier if we define names in this workbook as you did in the previous part of the exercise. Once again, make sure you are in the “**Part 2 – Reservoir**” sheet. Go to cell C54. Then, go to the menu of Formulas and click on “Define Name”. A dialogue window should come out, you should name the variable “River_WD” as you did before which as a short name of “River Water Demand”. Very important, make sure that in “Scope:” drop down menu you select “**Part 2 - Reservoir**”. By default the “Refers to:” section should be selecting the cell C54 where the cursor is located. If this is not showing, click on the “cells and arrow” icon on the right and select the cell C54. By doing this, you are naming that cell “River_WD” that you will be able to call it like this worksheet in excel, cool, isn’t it!

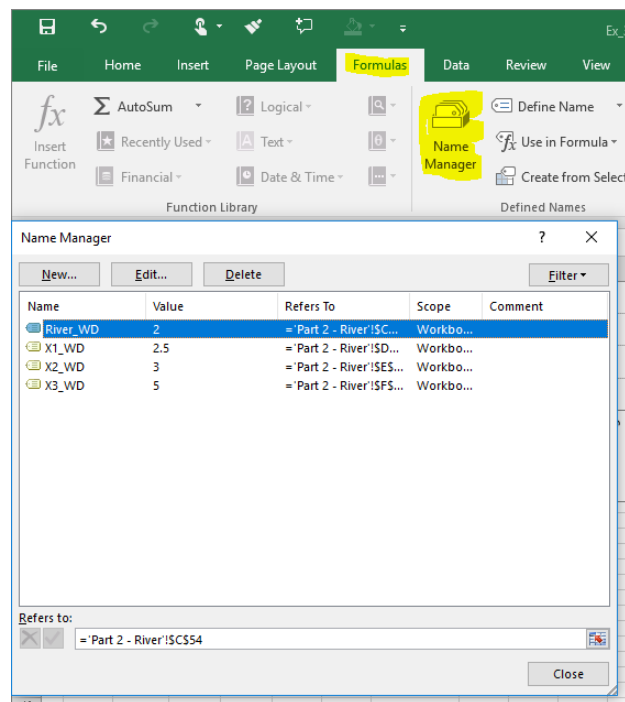


You will have to “Define Name” for the following variables and cells:

“Define Name”	Cell
Variable	
River_WD	C54

X1_WD	D54
X2_WD	E54
X3_WD	F54
K	J54
Hedging	K54

If you had a problem naming your variables or not selecting the right cell, you can click on the Formulas/Name Manager where you can delete or edit you variables



The same file has a system set up with a reservoir, with capacity **$K=25$ units**, and initial storage **$S_0=7$ units**, as shown below.

Water Demands				
Priority	1	3	3	2
Name	River	User 1	User 2	User 3

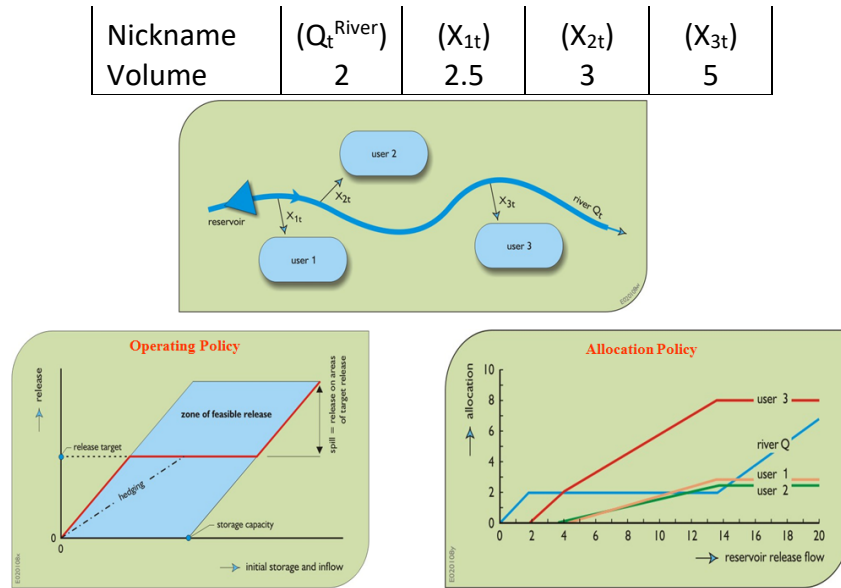


Figure 17

The only difference in the allocation policy is that for *User 1*, *User 2* and *User 3*, you will have to substitute $S_{t-1} + Q_t^{in}$ instead of Q_t^{in} in each of the conditionals. For River, Q_t^{River} , the water allocation changes as follow:

$$Q_t^{River} = \begin{cases} Q_t^{in} + S_{t-1} & \text{if } Q_t^{in} + S_{t-1} < 2 \\ 2 & \text{if } Q_t^{in} + S_{t-1} \geq 2 \end{cases}$$

In addition, reservoir spills must be calculated (Column G, Cells G6:G40) as follow:

$$Q_t^{Spill} = \begin{cases} 0 & \text{if } S_{t-1} + Q_t^{in} \leq K + Q_t^{River} + \sum_{i=1}^{i=3} X_{it} \\ (S_{t-1} + Q_t^{in}) - \left(K + Q_t^{River} + \sum_{i=1}^{i=3} X_{it} \right) & \text{if } S_{t-1} + Q_t^{in} > K + Q_t^{River} + \sum_{i=1}^{i=3} X_{it} \end{cases}$$

- 1) Please write down a paragraph explaining in plain words how the operation for River has changed, such as the one used in the River System:
 “River has the first priority, at least 2 units of water should be left in the river (if available) before the rest of the users can take their water demand. [Insert here how the water allocation policy has changed for River].”

Hint: In order for the river to receive extra water, does the reservoir have to spill?

- 2) Calculate the spills (Column G, Cells G6:G40) using the equation provided above.
- 3) Estimate the table for the water allocation policy (cells B2:H40) and show this table.
- 4) Estimate in column D the $S_{t-1} + Q_t^{\text{in}}$.
- 5) Use any of the two methods explained above to link the water allocation policy table with the mass balance model for the River (E59:E79), User 1 (F59:F79), User 2 (G59:G79) and User 3 (H59:H79).
- 6) Using the mass balance equation, estimate the reservoir storage through every time step (I59:I79). Remember that the storage can't exceed the storage capacity $K=25$ units.
- 7) Use any of the two methods explained above to link the water allocation policy table with the mass balance model for Reservoir spills (J59:J79),
- 8) Display the mass balance results, like figure 15 and 16.
- 9) Similarly, does the water supply for Users 1, 2 and 3 improved if the reservoir is built? Show a comparison of the water supply for both systems (With and without a reservoir $Q_t^{\text{River}} = 2 \text{ units}$ in both calculations)
- 10) What about the River? Does the river improve their water supply?
- 11) Show table (like figure 17) of the four performance criteria [Reliability(time and volume), vulnerability and resilience) and the Sustainability Index.
- 12) Did the performance of the systems improved or got worse? Create a table showing a comparison the performance criteria and Sustainability Index

Optimization Modeling

Objective

The objective of this exercise is to provide a set of problems using simple optimization techniques as well as the basics of linear programming using a linear optimization solver in Excel.

Useful Formulas

Find the decision variables, $\mathbf{x}$, that optimizes (maximizes or minimizes) an objective function.

For instance, to minimize:

$$\text{minimize } f(\mathbf{x})$$

← Objective function

$\mathbf{x}$

$$\text{subject to}$$

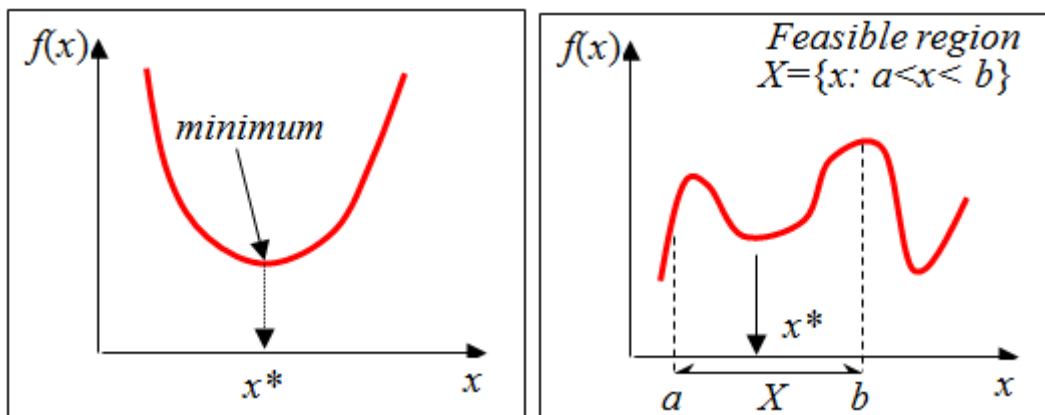
← Decision variables

$$\mathbf{x} \in X$$

$$Ax + b \leq c$$

← Constraint set

$$x \geq 0$$



Steps for Linear Programming

In order to solve linear programming, these are 7 steps that you should follow to identify the maximum or minimum value for the objective function at hand:

1. Define the *optimization purpose*. Is the objective to **Maximize or Minimize**?
2. Define *Objective Function*, or in other words, write the Objective Function into an equation. If the equation is linear ($Z=ax_1+bx_2$), it can be solved through a linear programming. If not, other techniques of non-linear programming can be used to solve this type of problem.
3. Define the Constraints. Write the constraints into inequalities, so they can be used to define the Feasible Region. Notice that the statement “the value of x is greater than (or at least) 20” means $x \geq 20$, and the statement “the value of x is smaller than (or less than) 20” means $x \leq 20$.
4. Define the Feasible Region. Use the constraints (inequalities) to bound the feasible region. For constraints with the form “ $ax_1+bx_2 \leq C$ ”, convert them into equations by dropping the “ $\leq$ ” or “ $\geq$ ” symbol and adding the equal sign “ $=$ ”, such as “ $ax_1+bx_2=C$ ”. Then solve the equation for $x_1=0$ to obtain the point where the linear equation crosses the x_1 axis. Similarly, solve for $x_2=0$ to obtain the point where the linear equation cross the x_2 axis.
5. Obtain the vertices of the feasible region. Do this by identifying the points in the feasible region and, for the constraints with the form “ $ax_1+bx_2 \leq C$ ”, convert them into equations “ $ax_1+bx_2=C$ ” and obtain the value of the unknown variable, either x_1 or x_2 , by solving the equation.
6. Substitute vertices into the Objective Function. Substitute the values of the variables (x_1 and x_2) at the vertices into the objective function equation.
7. Select the values of the variables that are the maximum or minimum for the objective function, depending on the definition of the objective function (Step 1).

Installing Open Solver - Model

Installing the Open Solver in excel.

1.- Copy the OpenSolver21.zip into your computer.

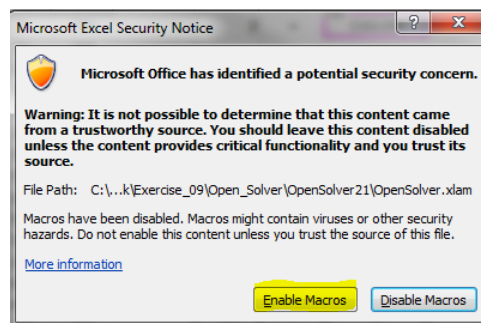
2.- Unzip this file

Name	Date modified	Type	Size
cbc	2/24/2012 8:45 PM	Application	2,913 KB
CPL License	7/7/2011 12:35 PM	Text Document	12 KB
GNU GPL License	7/7/2011 12:35 PM	Text Document	35 KB
OpenSolver ChangeLog	9/5/2012 2:45 PM	Microsoft Excel Worksheet	18 KB
OpenSolver	9/6/2012 10:06 PM	Microsoft Excel Add-In	467 KB
ReadMe	11/12/2011 6:55 AM	Text Document	5 KB

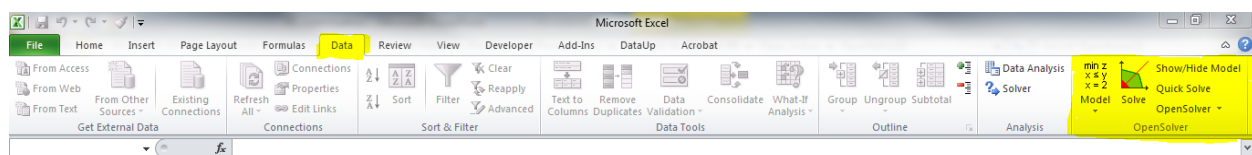
3.- Double click on the file: “OpenSolver” which is the add-In

Name	Date modified	Type	Size
cbc	2/24/2012 8:45 PM	Application	2,913 KB
CPL License	7/7/2011 12:35 PM	Text Document	12 KB
GNU GPL License	7/7/2011 12:35 PM	Text Document	35 KB
OpenSolver ChangeLog	9/5/2012 2:45 PM	Microsoft Excel Worksheet	18 KB
OpenSolver	9/6/2012 10:06 PM	Microsoft Excel Add-In	467 KB
ReadMe	11/12/2011 6:55 AM	Text Document	5 KB

4.- If a dialogue-window pops up, please select “enable macros”



5.- On the data menu of Excel you should see the Open Solver add-in in the right corner of the menu.



6.- Finally Open the file Ex_7.xlsx

For **Windows** user is pretty straight forward

1. Download the 'OpenSolver.zip file' from www.opensolver.org
2. Extract the files to a convenient location
3. Double click on 'OpenSolver.xlam'
4. If asked, give Excel permissions to run OpenSolver

For **MacOS** users you will have to do a couple of extra steps.

1. Download the 'OpenSolver.zip file' from www.opensolver.org
2. Extract the files to a convenient location
3. Double click on 'OpenSolver.xlam'
4. In the same folder, you will have to open another folder called "Solvers\osx" and then double click in the file "OpenSolver Solvers.pkg"
5. If a message says that This file cannot be verified and to delete the file or "done", then do not click on "Delete this file". Click in Done. And do the following steps:
6. Open the "System Settings" of your computer, then click on "Privacy and Security", then scroll down to the "Security" section, and in this section there should be a message stating that the "OpenSolver Solvers.pkg" package was blocked. Click on "Open Anyway"

Exercise 1 (Adapted from McKinney).

Based on an annual water allocation of 1,800 acre-feet (AF), an irrigation district wants to know how they can maximize their profits by growing two types of crops, Crop A and Crop B. The profits can be obtained by summing the profit (C_A and C_B) times the acreage (X_A and X_B) for each crop. The irrigation district has constraints. The first constraint is the water use, which is the sum of the water requirement times the acreage for each crop, which must be equal or less than the water allocation (1,800 AF). Second, there are limits to growing one crop or another. The maximum acreage on which to grow crop A is 400 acre, and crop B is 600 acres. Lastly, the acreage must be a positive number for both crops. Table 1 shows the water requirement, expected profit per acre and maximum area cultivated for each crop.

Table 1

	Crop A	Crop B
Water requirement (Acre feet/acre)	3	2
Profit (\$/acre)	300	500
Max area (acres)	400	600

To be turned in:

- 1) Write down the objective equations and constraints for this problem. Take a look at the “Optimization Presentation”, this exercise was explained in this presentation.
- 2) Copy the chart for the feasible region located in Ex_5.xlsx file tab “1 Maximize”
- 3) What was the solution (Maximum value for the objective function Z) in the presentation?

Now, let’s work on the Ex_5.xlsx file. Open the tab “1 Maximize”. The right side of that tab should look like Figure 1. Every cell that is orange means you must declare some information there.

Variables and Objective Function

First, let’s start with the *Variables*. We need to give the model a first guess of the variable, usually any value inside the feasible region and the model will take care to later give us the right result once we have run it. For X_A let’s define an initial value of 400 (cell T4=400), and for X_B a value of 600 (cell T5=600), see Figure 2.

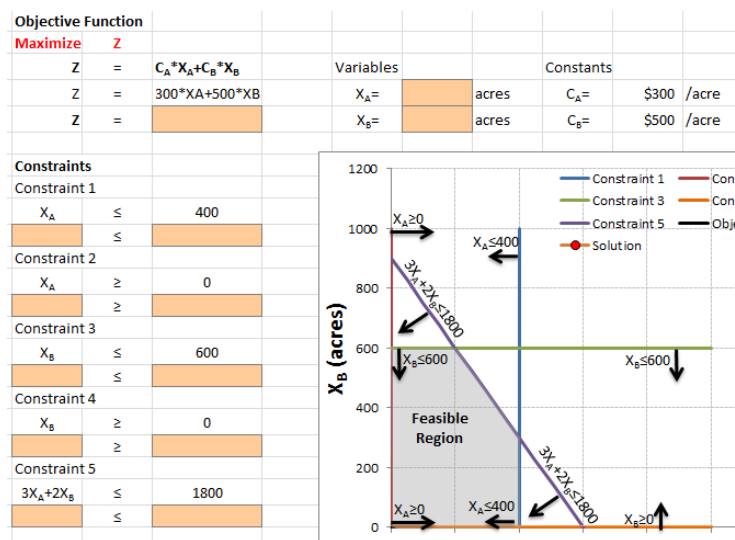


Figure 1

T4		f _x		400			
	R	S	T	U	V	W	X
1							
2							
3		Variables		Constants			
4		X_A	400	acres	C_A	\$300	/acre
5		X_B	600	acres	C_B	\$500	/acre

Figure 2

Now let's define the Objective function in cell Q5 as the sum of the multiplication of the acreage times the profits for each crop (cell Q5 = T4*W4+T5*W5), see Figure 3.

TREND		X ✓ f _x		=T4*W4+T5*W5						
	O	P	Q	R	S	T	U	V	W	X
1	Objective Function									
2	Maximize	Z			Variables			Constants		
3	Z	=	C _A *X _A +C _B *X _B		Acreage			Profit		
4	Z	=	300*X _A +500*X _B		X _A =	400	acres	C _A =	\$300	/acre
5	Z	=	=T4*W4+T5*W5		X _B =	600	acres	C _B =	\$500	/acre

Figure 3

Constraints

Now let's work on the *Constraints*. Constraint 1 specifies that the acreage of crop a (X_A) must be equal to or less than 400 acres. Link cell O10 with the value of X_A (cell T4) by typing in cell O10 "=T4"; see Figure 4. Type in cell Q10 the constraint value of 400 (Figure 5).

TREND									
	M	N	O	P	Q	R	S	T	U
1			Objective Function						
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	$C_A * X_A + C_B * X_B$		Acreage		
4	XB		Z	=	$300 * X_A + 500 * X_B$		$X_A =$	400	acres
5	840		Z	=	\$420,000		$X_B =$	600	acres
6	810								
7	780		Constraints						
8	750		Constraint 1						
9	720		X_A	≤	400				
10	690		=T4	≤					

Figure 4

Q10									
	M	N	O	P	Q	R	S	T	U
1			Objective Function						
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	$C_A * X_A + C_B * X_B$		Acreage		
4	XB		Z	=	$300 * X_A + 500 * X_B$		$X_A =$	400	acres
5	840		Z	=	\$420,000		$X_B =$	600	acres
6	810								
7	780		Constraints						
8	750		Constraint 1						
9	720		X_A	≤	400				
10	690		400	≤	400				

Figure 5

Constraint 2 specifies that the acreage of crop A (X_A) must be equal to or greater than 0 acres. Link cell O13 with the value of X_A (cell T4) by typing in cell O13 “=T4”; see Figure 6. Type in cell Q13 the constraint value of 0 (Figure 7).

TREND									
	M	N	O	P	Q	R	S	T	U
1			Objective Function						
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	$C_A * X_A + C_B * X_B$		Acreage		
4	XB		Z	=	$300 * X_A + 500 * X_B$		$X_A =$	400	acres
5	840		Z	=	\$420,000		$X_B =$	600	acres
6	810								
7	780		Constraints						
8	750		Constraint 1						
9	720		X_A	≤	400				
10	690		400	≤	400				
11	660		Constraint 2						
12	630		X_A	≥	0				
13	600		=T4	≥					

Figure 6

Q13									
	M	N	O	P	Q	R	S	T	U
1			Objective Function						
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	$C_A * X_A + C_B * X_B$		Acreage		
4	XB		Z	=	$300 * X_A + 500 * X_B$		$X_A =$	400	acres
5	840		Z	=	\$420,000		$X_B =$	600	acres
6	810								
7	780		Constraints						
8	750		Constraint 1						
9	720		X_A	≤	400				
10	690		400	≤	400				
11	660		Constraint 2						
12	630		X_A	≥	0				
13	600		400	≥	0				

Figure 7

Constraint 3 specifies that the acreage of crop B (X_B) must be equal to or less than 600 acres. Link cell O16 with the value of X_B (cell T5) by typing in cell O16 “=T5”; see Figure 8. Type in cell Q16 the constraint value of 600 (Figure 9).

TREND X ✓ fx =T5									
	M	N	O	P	Q	R	S	T	U
1			Objective Function						
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	$C_A * X_A + C_B * X_B$		Acreage		
4	XB		Z	=	$300 * X_A + 500 * X_B$		$X_A =$	400	acres
5	840		Z	=	\$420,000		$X_B =$	600	acres
6	810								
7	780		Constraints						
8	750		Constraint 1						
9	720		X_A	≤	400				
10	690		400	≤	400				
11	660		Constraint 2						
12	630		X_A	≥	0				
13	600		400	≥	0				
14	570		Constraint 3						
15	540		X_B	≤	600				
16	510		=T5	≤					

Figure 8

Q16 X ✓ fx 600									
	M	N	O	P	Q	R	S	T	U
1			Objective Function						
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	$C_A * X_A + C_B * X_B$		Acreage		
4	XB		Z	=	$300 * X_A + 500 * X_B$		$X_A =$	400	acres
5	840		Z	=	\$420,000		$X_B =$	600	acres
6	810								
7	780		Constraints						
8	750		Constraint 1						
9	720		X_A	≤	400				
10	690		400	≤	400				
11	660		Constraint 2						
12	630		X_A	≥	0				
13	600		400	≥	0				
14	570		Constraint 3						
15	540		X_B	≤	600				
16	510		600	≤	600				

Figure 9

Constraint 4 specifies that the acreage of crop B (X_B) must be equal or greater than 0 acres. Link cell O19 with the value of X_B (cell T5) by typing in cell O19 “=T5”; see Figure 10. Type in cell Q19 the constraint value of 0 (Figure 11).

TREND									
	M	N	O	P	Q	R	S	T	U
1			Objective Function						
2	Active		Maximize	Z			Variables		
3	500*XB		Z	=	$C_A * X_A + C_B * X_B$		Acreage		
4	XB		Z	=	$300 * X_A + 500 * X_B$		$X_A =$	400	acres
5	840		Z	=	\$420,000		$X_B =$	600	acres
6	810								
7	780		Constraints						
8	750		Constraint 1						
9	720		X_A	$\leq$	400				
10	690		400	$\leq$	400				
11	660		Constraint 2						
12	630		X_A	$\geq$	0				
13	600		400	$\geq$	0				
14	570		Constraint 3						
15	540		X_B	$\leq$	600				
16	510		600	$\leq$	600				
17	480		Constraint 4						
18	450		X_B	$\geq$	0				
19	420		=T5	$\geq$					

Figure 10

Q19									
	M	N	O	P	Q	R	S	T	U
1			Objective Function						
2	Active		Maximize	Z			Variables		
3	500*XB		Z	=	$C_A * X_A + C_B * X_B$		Acreage		
4	XB		Z	=	$300 * X_A + 500 * X_B$		$X_A =$	400	acres
5	840		Z	=	\$420,000		$X_B =$	600	acres
6	810								
7	780		Constraints						
8	750		Constraint 1						
9	720		X_A	$\leq$	400				
10	690		400	$\leq$	400				
11	660		Constraint 2						
12	630		X_A	$\geq$	0				
13	600		400	$\geq$	0				
14	570		Constraint 3						
15	540		X_B	$\leq$	600				
16	510		600	$\leq$	600				
17	480		Constraint 4						
18	450		X_B	$\geq$	0				
19	420		600	$\geq$	0				

Figure 11

Constraint 5 specifies that the sum of the water use in crop A ($3 * X_A$) and crop B ($2 * X_B$) must be equal or less than 1800 acre-feet. In cell O22 we have to write this equation as follow: “=Z4*T4+Z5*T5”; see Figure 12. Type in cell Q19 the constraint value of 1800 (Figure 13).

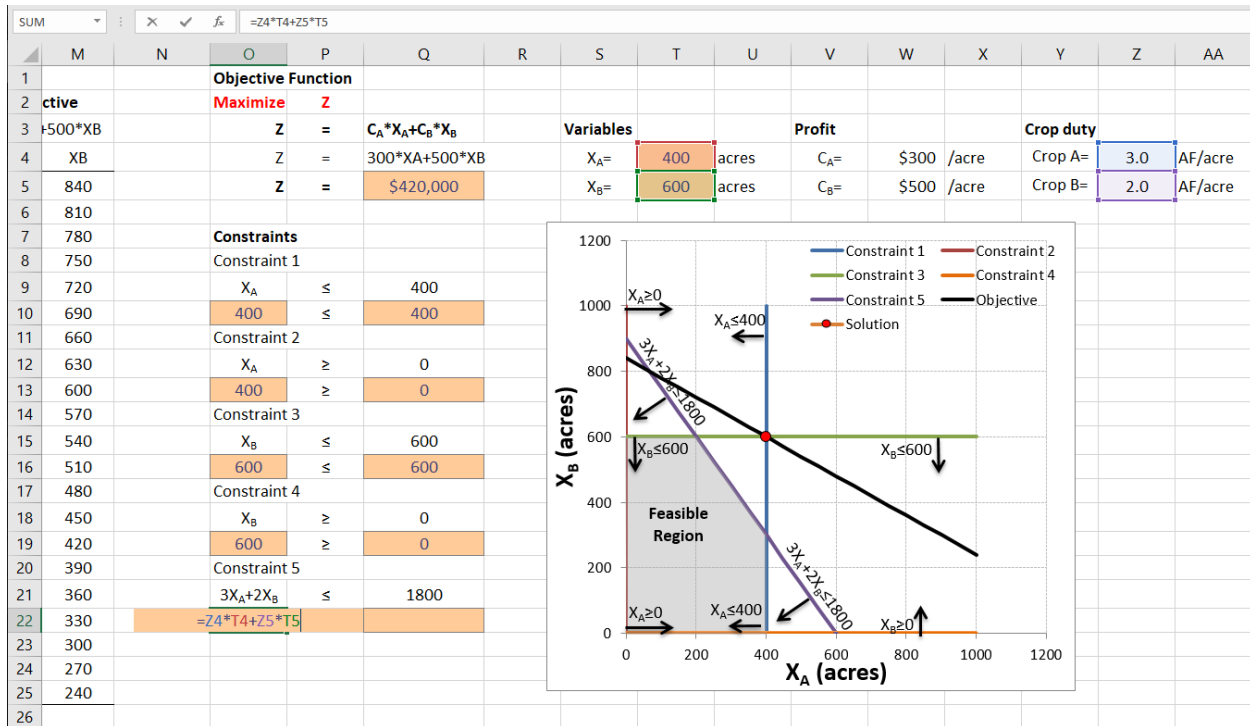


Figure 12

Q22				1800					
	M	N	O	P	Q	R	S	T	U
1			Objective Function						
2	Active		Maximize	Z			Variables		
3				Z	=	$C_A * X_A + C_B * X_B$	Acreage		
4				Z	=	$300 * X_A + 500 * X_B$	$X_A =$	400	acres
5				Z	=	\$420,000	$X_B =$	600	acres
6	810								
7	780								
8	750								
9	720								
10	690								
11	660								
12	630								
13	600								
14	570								
15	540								
16	510								
17	480								
18	450								
19	420								
20	390								
21	360								
22	330								

Figure 13

Save your spreadsheet (Ctrl+S).

Defining the Optimization Linear Program

Now, let's define the linear program model. Go to the Data menu and click on “Min Z $x \leq y$ $x=2$ Model” icon (Figure 14)

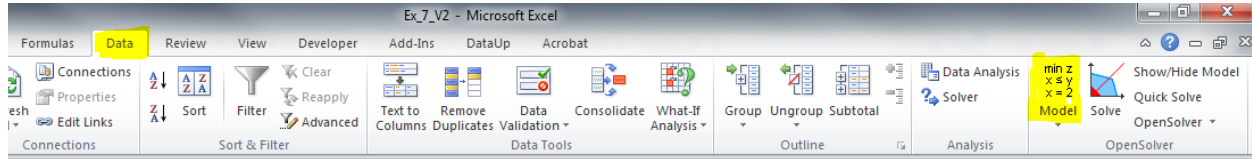


Figure 14

A dialogue window will appear, displaying the “Open Solver - Model” (Figure 15).

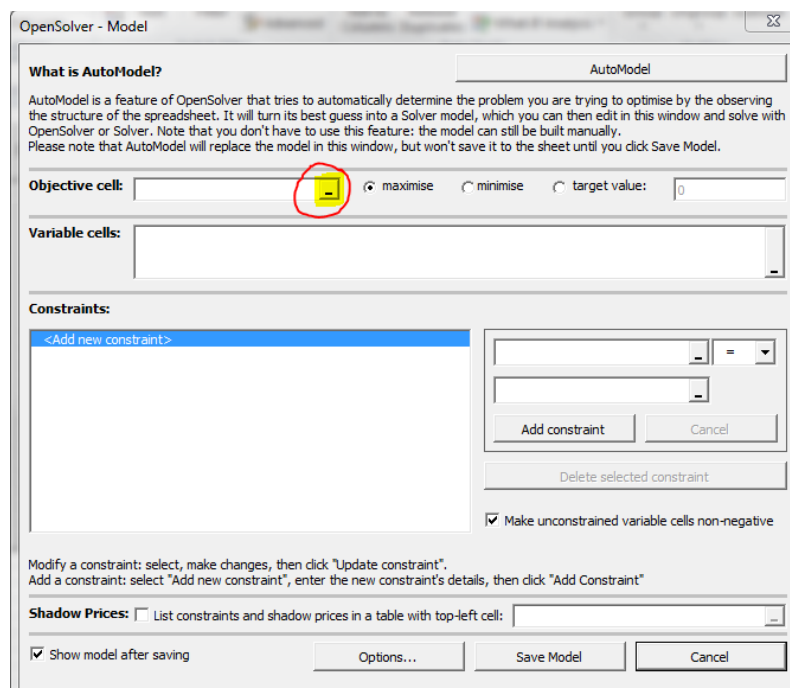


Figure 15

Go to the Objective cell section, and click on the icon to browse the objective cell (Figure 15). Select the cell Q5, which is the cell with the equation of the objective function (Figure 16).

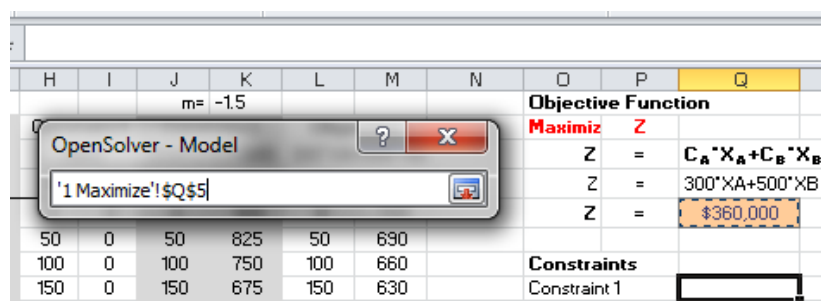


Figure 16

When you come back to the dialogue window, make sure to select “maximize” (Figure 17).

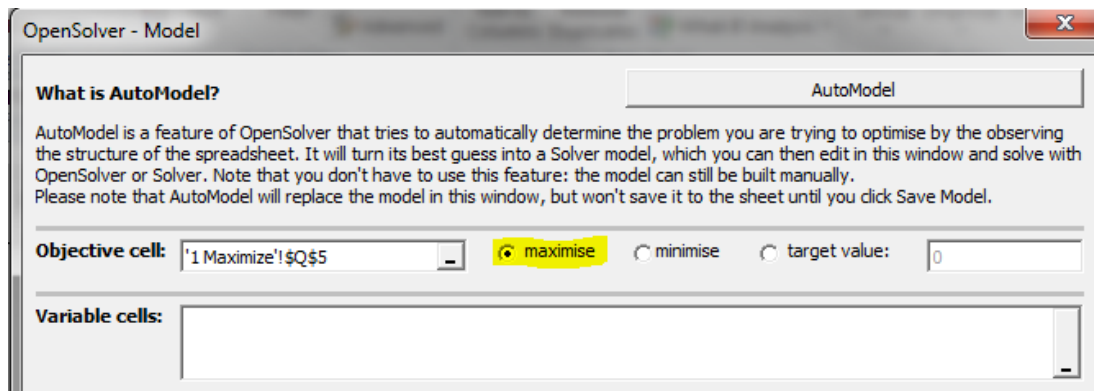


Figure 17

Now click on the icon to choose the cells that will be the variables (Figure 18). Select cells T4 and T5 (Figure 19).

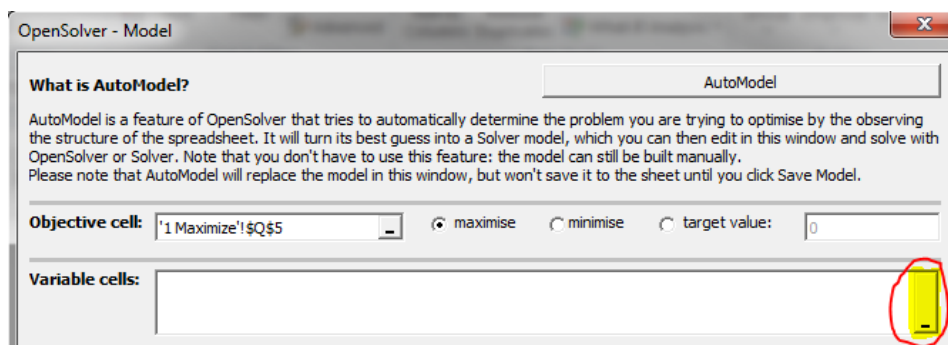


Figure 18

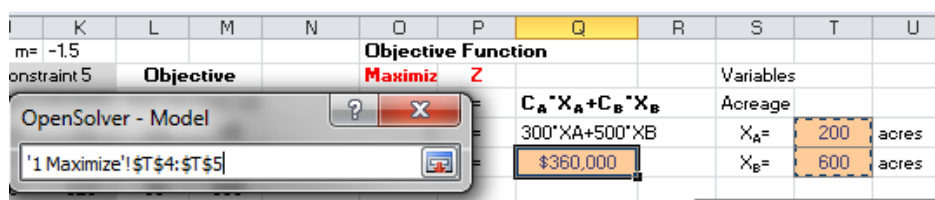


Figure 19

Now, we will add the constraints to the model. Let's start with the first constraint ($X_A \leq 400$), go to the constraints section and click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O10 (Figure 21), then go back to the dialogue menu. Select the symbol of equal or less than " $\leq$ " (Figure 22). Now click on the icon to select the right side of the inequality (Figure 23), cell Q10 (Figure 24) and go back the dialogue window. Click on "Add Constraint" (Figure 25). The inequality Cell O10 $\leq$ Cell Q10 should have appeared in the left side of the Constraints section of the dialogue window (Figure 26). We just have finished declaring the first constraint.

OpenSolver - Model

What is AutoModel? AutoModel

AutoModel is a feature of OpenSolver that tries to automatically determine the problem you are trying to optimise by observing the structure of the spreadsheet. It will turn its best guess into a Solver model, which you can then edit in this window and solve with OpenSolver or Solver. Note that you don't have to use this feature: the model can still be built manually. Please note that AutoModel will replace the model in this window, but won't save it to the sheet until you click Save Model.

Objective cell: '1 Maximize'!\$Q\$5 maximise minimise target value: 0

Variable cells: '1 Maximize'!\$T\$4:\$T\$5

Constraints:

<Add new constraint>

'1 Maximize'!\$O\$10 = < >

Add constraint Cancel

Delete selected constraint

Figure 23

H	I	J	K	L	M	N	O	P	Q
		m= -1.5					Objective Function		
Constraint 4		Constraint 5		Objective			Maximize	Z	
	XB=0	3XA+2XB≤1800		300*XA+500*XB			Z =	$C_A * X_A + C_B * X_B$	
	XA XB	XA	XB	XA	XB		Z =	300*XA+500*XB	
	0 0	0	300	0	720		Z =	\$360,000	
	50 0	50	825	50	690				

OpenSolver - Model		
'1 Maximize'!\$Q\$10		

Constraints		
Constraint 1	XA	≤ 400
Constraint 2	200	≤ 400

Figure 24

OpenSolver - Model

What is AutoModel? AutoModel

AutoModel is a feature of OpenSolver that tries to automatically determine the problem you are trying to optimise by observing the structure of the spreadsheet. It will turn its best guess into a Solver model, which you can then edit in this window and solve with OpenSolver or Solver. Note that you don't have to use this feature: the model can still be built manually. Please note that AutoModel will replace the model in this window, but won't save it to the sheet until you click Save Model.

Objective cell: '1 Maximize'!\$Q\$5 maximise minimise target value: 0

Variable cells: '1 Maximize'!\$T\$4:\$T\$5

Constraints:

<Add new constraint>

'1 Maximize'!\$O\$10 = < >

'1 Maximize'!\$Q\$10 = < >

Add constraint Cancel

Figure 25

H	I	J	K	L	M	N	O	P	Q
		m= -1.5					Objective Function		
Constraint 4		Constraint 5		Objective			Maximiz	Z	
XB=0		3XA+2XB=1800		300*XA+500*XB			Z =	C _A *X _A +C _B *X _B	
XA	XB	XA	XB	XA	XB		Z =	300*XA+500*XB	
0	0	0	900	0	720		Z =	\$360,000	
50	0	50	825	50	630				
100	0	100	750	100	660				
150	0	150	675	150	630				
200	0	200	600	200	600				
250	0	250	525	250	570				
300	0	300	450	300	540				
350	0	350	375	350	510				
400	0	400	300	400	480				
							Constraints		
							Constraint 1		
							X _A	≤	400
							200	≤	400
							Constraint 2		
							X _A	≥	0
							200	≥	0
							Constraint 3		
							X _B	≤	600
							600	≤	600
							Constraint 4		

Figure 31

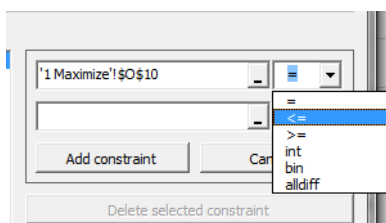


Figure 32

I	J	K	L	M	N	O	P	Q
m= -1.5						Objective Function		
Constraint 4		Constraint 5		Objective		Maximize Z		
XB=0		3XA+2XB=1800		300*XA+500*XB		Z = CA*XA+CB*XB		
XA	XB	XA	XB	XA	XB	Z = 300*XA+500*XB		
0	0	0	900	0	720	Z = \$360,000		
50	0	50	825	50	630			
100	0	100	750	100	660			
150	0	150	675	150	630			
200	0	200	600	200	600			
250	0	250	525	250	570			
300	0	300	450	300	540			
350	0	350	375	350	510			
400	0	400	300	400	480			
						Constraints		
						Constraint 1		
						XA	≤	400
						200	≤	400
						Constraint 2		
						XA	≥	0
						200	≥	0
						Constraint 3		
						XB	≤	600
						600	≤	600
						Constraint 4		

OpenSolver - Model

1 Maximize \$Q\$16

Figure 33

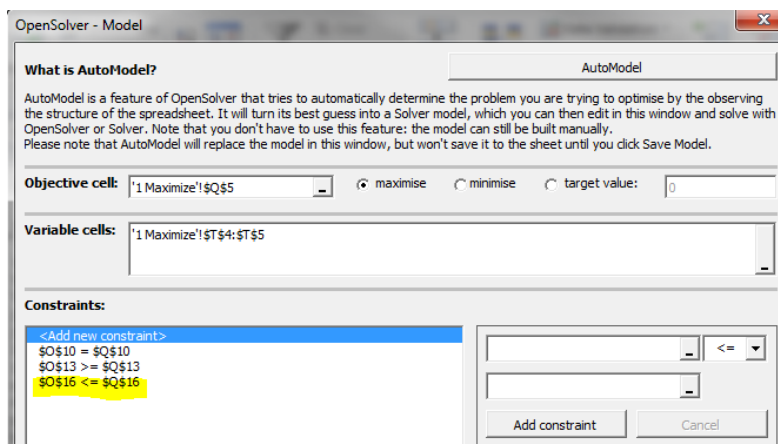


Figure 34

To declare the fourth constraint ($X_B \geq 0$), click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O19 (Figure 35), then go back to the dialogue menu. Select the symbol of equal or greater than “ $\geq$ ” (Figure 36). Now click on the icon to select right-side part of the inequality (Figure 23), select cell Q19 (Figure 37) and go back the dialogue window. Click on “Add Constraint”. The inequality Cell O19 $\geq$ Cell Q19 should have appeared in the left side of the Constraints section of the dialogue window (Figure 38).

H	I	J	K	L	M	N	O	P	Q
		m= -1.5							
Constraint 4		Constraint 5		Objective		Objective Function			
XB=0		3XA+2XB=1800		300*XA+500*XB		Maximize Z			
XA	XB	XA	XB	XA	XB	Z = C _A *XA+C _B *XB			
0	0	0	900	0	720	Z = 300*XA+500*XB			
50	0	50	825	50	690				
100	0	100	750	100	660				
150	0	150	675	150	630				
200	0	200	600	200	600				
250	0	250	525	250	570				
300	0	300	450	300	540				
350	0	350	375	350	510				
400	0	400	300	400	480				
450	0	450	225	450	450				
500	0	500	150	500	420				
550	0	550	75	550	390				

OpenSolver - Model

?

X

'1 Maximize'!\$O\$19

Constraints	
Constraint 1	
XA ≤ 400	
200 ≤ 400	
Constraint 2	
XA ≥ 0	
200 ≥ 0	
Constraint 3	
XB ≤ 600	
600 ≤ 600	
Constraint 4	
XB ≥ 0	
600 ≥ 0	
Constraint 5	

Figure 35

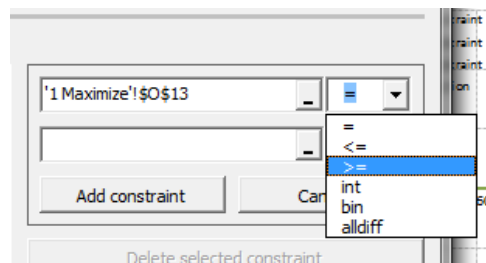


Figure 36

H	I	J	K	L	M	N	O	P	Q
		m= -1.5							
Constraint 4		Constraint 5		Objective		Objective Function			
XB=0		3XA+2XB=1800		300*XA+500*XB		Maximize Z		C _A *X _A +C _B *X _B	
XA	XB	XA	XB	XA	XB	Z =		300*XA+500*XB	
0	0	0	900	0	720	Z =		\$360,000	
50	0	50	825	50	690				
100	0	100	750	100	660				
150	0	150	675	150	630				
200	0	200	600	200	600				
250	0	250	525	250	570				
300	0	300	450	300	540				
350	0	350	375	350	510				
400	0	400	300	400	480				
450	0	450	225	450	450				
500	0	500	150	500	420				

OpenSolver - Model

?

X

'1 Maximize '\$Q\$19'

Constraints	
Constraint 1	
X _A	≤ 400
200	≤ 400
Constraint 2	
X _A	≥ 0
200	≥ 0
Constraint 3	
X _B	≤ 600
600	≤ 600
Constraint 4	
X _B	≥ 0
600	≥ 0

Figure 37

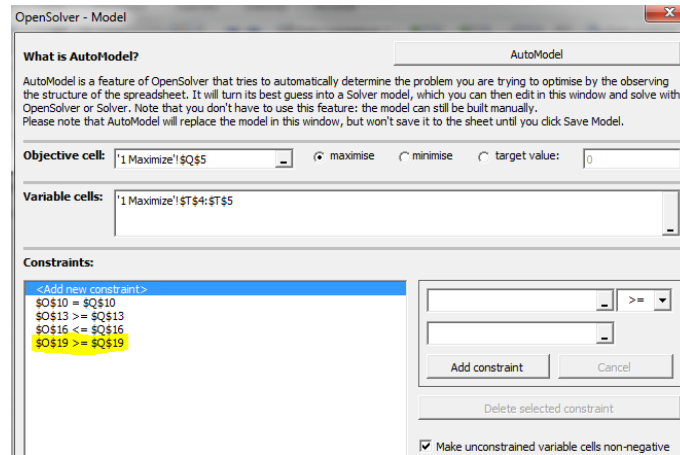


Figure 38

To declare the fifth constraint ($3X_A + 2X_B \leq 1800$), click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O22 (Figure 39), then go back to the dialogue menu. Select the symbol of equal or greater than “ $\leq$ ” (Figure 40). Now click on the icon to select right-side part of the inequality (Figure 23), select cell Q22 (Figure 41) and go back the dialogue window. Click on “Add Constraint”. The inequality of cell $O22 \geq$ Cell Q22 should have appeared in the left side of the Constraints section of the dialogue window (Figure 42).

H	I	J	K	L	M	N	O	P	Q
		m= -1.5					Objective Function		
Constraint 4		Constraint 5		Objective			Maximize	Z	
XB ≥ 0		3XA + 2XB ≤ 1800		300*XA + 500*XB			Z =	C _A *X _A + C _B *X _B	
XA	XB	XA	XB	XA	XB		Z =	300*XA + 500*XB	
0	0	0	900	0	720		Z =	\$360,000	
50	0	50	825	50	690		Constraints		
100	0	100	750	100	660		Constraint 1		
150	0	150	675	150	630		X _A	≤	400
200	0	200	600	200	600		200	≤	400
250	0	250	525	250	570		Constraint 2		
300	0	300	450	300	540		X _A	≥	0
350	0	350	375	350	510		200	≥	0
400	0	400	300	400	480		Constraint 3		
450	0	450	225	450	450		X _B	≤	600
500	0	500	150	500	420		600	≤	600
550	0	550	75	550	390		Constraint 4		
600	0	600	0	600	360		X _B	≥	0
650	0	650	-75	650	330		600	≥	0
700	0	700	-150	700	300		Constraint 5		
							3XA + 2XB	≤	1800
							1800	≤	1800

Figure 39

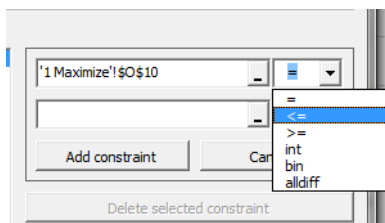


Figure 40

	I	J	K	L	M	N	O	P	Q
		m= -1.5					Objective Function		
Constraint 4		Constraint 5		Objective			Maximize	Z	
XB=0		3XA+2XB=1800		300*XA+500*XB			Z =	$C_A \cdot X_A + C_B \cdot X_B$	
XA	XB	XA	XB	XA	XB		Z =	300*XA+500*XB	
J	0	0	900	0	720		Z =	\$360,000	
10	0	50	825	50	690				
10	0	100	750	100	660		Constraints		
10	0	150	675	150	630		Constraint 1		
10	0	200	600	200	600		X_A	$\leq$	400
10	0	250	525	250	570		200	$\leq$	400
10	0	300	450	300	540		Constraint 2		
10	0	350	375	350	510		X_A	$\geq$	0
10	0	400	300	400	480		200	$\geq$	0
10	0	450	225	450	450		Constraint 3		
10	0	500	150	500	420		X_B	$\leq$	600
10	0	550	75	550	390		600	$\leq$	600
10	0	600	0	600	360		Constraint 4		
10	0	650	-75	650	330		X_B	$\geq$	0
							600	$\geq$	0
							Constraint 5		
							$3X_A + 2X_B$	$\leq$	1800
							1800	$\leq$	1800

Figure 41

OpenSolver - Model

What is AutoModel? AutoModel

AutoModel is a feature of OpenSolver that tries to automatically determine the problem you are trying to optimise by the observing the structure of the spreadsheet. It will turn its best guess into a Solver model, which you can then edit in this window and solve with OpenSolver or Solver. Note that you don't have to use this feature: the model can still be built manually. Please note that AutoModel will replace the model in this window, but won't save it to the sheet until you click Save Model.

Objective cell: ☒ maximise ☐ minimise ☐ target value:

Variable cells:

Constraints:

<Add new constraint>

\$Q\$10 <= \$Q\$10
 \$Q\$13 >= \$Q\$13
 \$Q\$16 <= \$Q\$16
 \$Q\$19 >= \$Q\$19
 \$Q\$22 <= \$Q\$22

<=

Update constraint Cancel

Delete selected constraint

☒ Make unconstrained variable cells non-negative

Figure 42

Check the box that says “List constraints and shadow prices in a table with top left cell:” and select the cell “AC8” (Figure 43)

OpenSolver - Model

What is AutoModel? AutoModel

AutoModel is a feature of OpenSolver that tries to automatically determine the problem you are trying to optimise by the observing the structure of the spreadsheet. It will turn its best guess into a Solver model, which you can then edit in this window and solve with OpenSolver or Solver. Note that you don't have to use this feature: the model can still be built manually. Please note that AutoModel will replace the model in this window, but won't save it to the sheet until you click Save Model.

Objective cell: ☒ maximise ☐ minimise ☐ target value:

Variable cells:

Constraints:

- <Add new constraint>
- \$O\$10 <= \$Q\$10
- \$O\$13 >= \$Q\$13
- \$O\$16 <= \$Q\$16
- \$O\$19 >= \$Q\$19
- \$O\$22 <= \$Q\$22

<=

☒ Make unconstrained variable cells non-negative

Modify a constraint: select, make changes, then click "Update constraint".
Add a constraint: select "Add new constraint", enter the new constraint's details, then click "Add Constraint"

Shadow Prices: ☒ List constraints and shadow prices in a table with top-left cell:

☒ Show model after saving

Figure 43

Click on “Save Model” (Figure 44). A Series of colorful rectangles should have appeared in your Spreadsheet (Figure 45). Notice that the Objective Function (cell Q5) has a “Max” label on top of it. Also, notice that the variables have a pink rectangle on top. In addition, for each constraint the right and left side of the equations/inequality are related with a line and a symbol (“ $\geq$ ”, “ $\leq$ ” or “ $=$ ”), Double check that these symbols are correct.

OpenSolver - Model

What is AutoModel? AutoModel

AutoModel is a feature of OpenSolver that tries to automatically determine the problem you are trying to optimise by the observing the structure of the spreadsheet. It will turn its best guess into a Solver model, which you can then edit in this window and solve with OpenSolver or Solver. Note that you don't have to use this feature: the model can still be built manually. Please note that AutoModel will replace the model in this window, but won't save it to the sheet until you click Save Model.

Objective cell: ☒ maximise ☐ minimise ☐ target value:

Variable cells:

Constraints:

- <Add new constraint>
- \$O\$10 = \$Q\$10
- \$O\$13 >= \$Q\$13
- \$O\$16 <= \$Q\$16
- \$O\$19 >= \$Q\$19
- \$O\$22 >= \$Q\$22

>=

☒ Make unconstrained variable cells non-negative

Modify a constraint: select, make changes, then click "Update constraint".
Add a constraint: select "Add new constraint", enter the new constraint's details, then click "Add Constraint"

Shadow Prices: ☐ List constraints and shadow prices in a table with top-left cell:

☒ Show model after saving

Figure 44

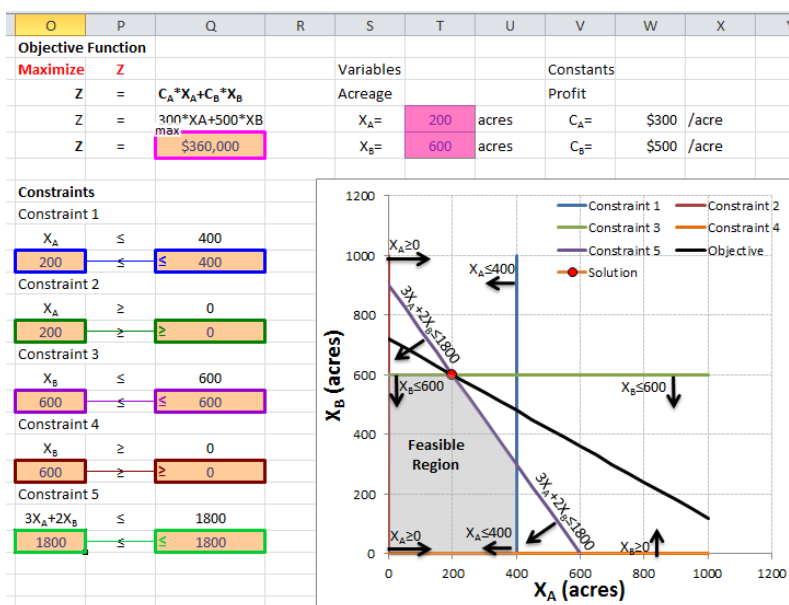


Figure 45

Running the Linear Programming Model

It is time to run the model!!! Click on the Solve icon (Figure 46) of the Open Solver. After the screen blinks you will notice that numbers have changed. The maximum profit that can be obtained are **\$360,000 (Cell Q5)!!!** The values for the variable have changed, $X_A=200$ and $X_B=600$, as we calculated in the presentation at class!!!

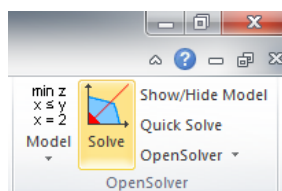


Figure 46

To be turned in:

- 1) Take a look at the shadow values. For the constraint in cells O16<=Q16 ($X_B \leq 600$) the shadow value is equal to -300, what does it mean? *Do the following to get a hint for the answer of this question:* Change the value of cell Q16 to 601 and run the model. What is the new value of the objective function (cell Q5)? Now change it to 602, and take a look at the objective function value. How much the objective value changed for 601? And for 602? And for 603? Does the shadow price tell you how much the objective function will change by a unit increase in the constraint?
- 2) A screenshot of the model, like Figure 45.
- 3) Now let's invert the values so the Profits for C_A (cell W4) are equal to \$500/acre and for C_B (cell W5) are equal to \$300/acre. Click on Solve. What are the results for X_A and X_B ?

Does the value of the profits change the results for the optimal solution? If so, do you think changing the market prices can change the optimal solution of linear systems?

- 4) A screenshot of the new model with the profit values changed.

Exercise 2 (Adapted from Loucks and van Beek)

A city planner wants to know the minimum cost of waste removal (WR) at Sites 1 and 2 upstream of a recreational park (Figure 47). The cost of waste removal can be obtained by summing the cost of 100% (C_1 and C_2) times the fraction of removal for each site (X_1 and X_2). According to a pre-design, the fraction of WR at site 1 (X_1) must be equal to or greater than 0.8 but equal to or less than 1. The fraction of WR at site 2 (X_2) must be equal to or less than 1. There is another constraint expressed as the equation: $X_1 + 1.3 X_2 \geq 1.8$.

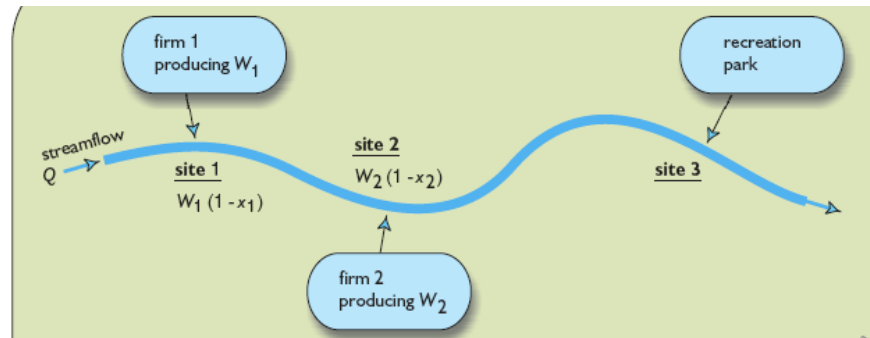


Figure 47.

Use the “Optimization” presentation in class to solve this problem. The optimization model is as follows:

Minimize Z

$$Z = C_1(x_1) + C_2(x_2)$$

Where: $C_1 = 200$; $C_2 = 100$

Subject to

$$x_1 \geq 0.8$$

$$x_1 \leq 1$$

$$x_2 \geq 0$$

$$x_2 \leq 1$$

$$x_1 + 1.3x_2 \geq 1.8$$

Using the Excel spreadsheet Ex_5.xlsx in Tab “2) Minimize”, create a linear program using the Open Solver Model. Don’t forget to select “**Minimize**” instead of “Maximize” (Figure 17) when creating the linear model.

To be turned in:

- 1) A screenshot of your linear model, such as Figure 44.
- 2) What is the minimum cost of removal (the optimum value of the objective function) obtained once the model has been run? How do your results compare with the ones in the presentation (Slide 20, minimum cost of removal equal to \$238K)

Exercise 3 The perfect outfit! Extra credits (+20%)

Summer is coming and you are looking for the perfect outfit to wear, while, at the trying not to over spend. You have a total budget of \$130. This is a suggested number of minimum and maximum number of pieces that you want to buy for each article.

Article	Minimum # of pieces	Maximum # of Pieces
Jeans (C_{Jeans})	1	3
T-Shirts ($C_{T-shirts}$)	1	4

Other constraints are that you want to buy equal or more T-shirts than jeans ($C_{T-shirts} \geq C_{Jeans}$), and that the total amount spent should not be more than your dedicated budget \$130.

To be turned in:

Following the seven steps of linear programming (page 4 of this exercise)

- 1) Write down the *objective function* and the constraints
- 2) Draw the *feasible region*. You can use excel, or a simple piece of paper and scan it, or some engineering paper.
- 3) Obtain the vertices of the feasible region. Substitute the values of the vertices into the objective function. Submit a table showing the vertices of the feasible region as well as the value of the objective function for each pair of C_{Jeans} and $C_{T-shirts}$ values (see slides 13 and 19 of the presentation of “Optimization”).
- 4) Based on the previous analysis, what is the combination Jeans and T-shirts that you should buy?

Now, let's use the Open Solver model of excel to solve this problem. Use the file Ex_5.xlsx - tab “3 Maximize” to create a linear optimization model. You can use as initial values for C_{Jeans} (cell G5) and $C_{T-shirt}$ (cell G6) 1 and 1, respectively. Calculate amount spend in cell D6 (for the objective function) and B23 (for the constraint) by multiplying the cost of the jeans times the number of jeans (J5xG5) plus the cost of T-shirts times the number of T-shirts (J6xG6). When creating the linear optimization model in Open Solver, don't forget to select “Maximize” (Figure 17). In addition, you will have to declare in the optimization model that the number of jeans C_{Jeans} (cell G5) and T-shirts $C_{T-shirts}$ (cell G6) should be integers (see figure 48).

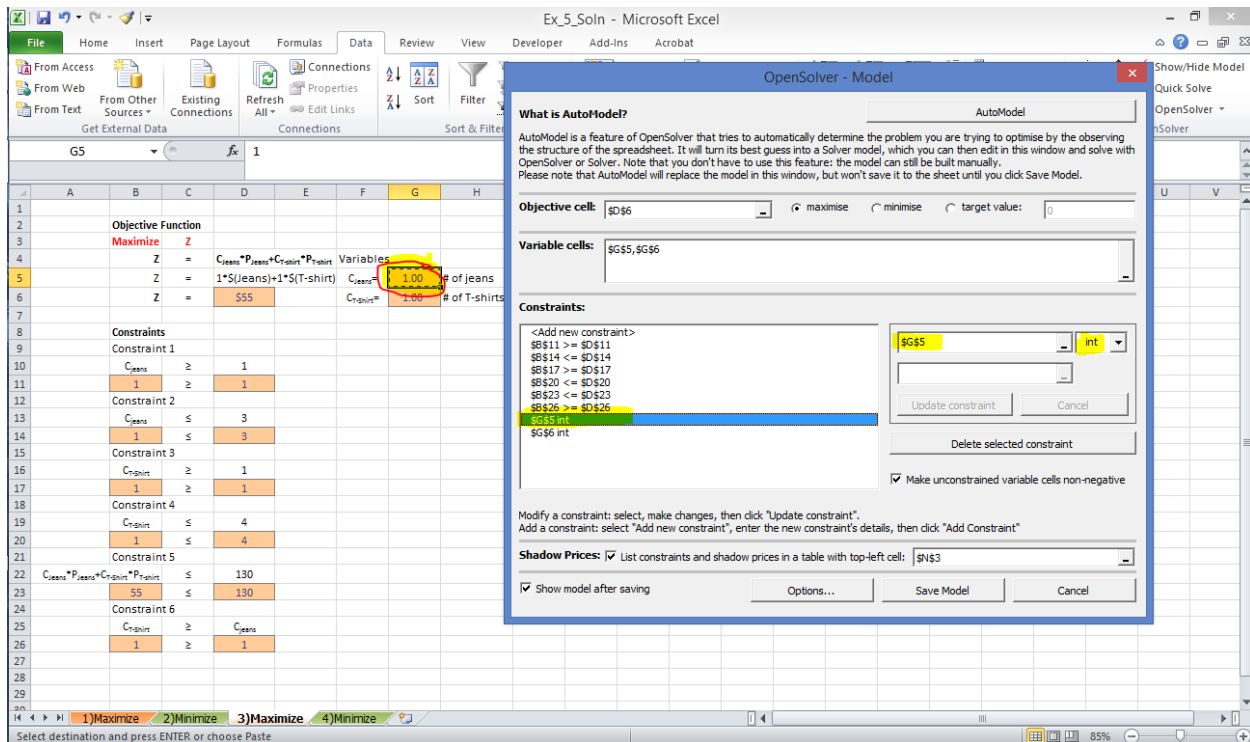


Figure 48

- 5) Turn in a screenshot of your linear model, similar to Figure 44.
- 6) How much will you spend (the optimum value of the objective function) obtained once the model has been run (cell D5)?
- 7) How many Jeans and T-shirts you can buy C_{Jeans} (cell G5) and $C_{T-shirts}$ (cell G6) for the maximum amount of money spent given the constraints?
- 8) How do these results compare with the same analysis done before in question 4 of this problem (see above)? Are these results similar?
- 9) What are the shadow prices for the different constraints? What is the maximum shadow price? If you change any constraint "1 unit", how much will the Objective function will change? Are there any binding constraints?

Exercise 4 (Adapted from Loucks and van Beek) Extra credits (+20%)

Consider a water-using industry that plans to obtain water from a groundwater aquifer. Two wellfield sites have been identified, *A* and *B*. The objective of this industry is to minimize the cost of pumping from the wells at sites *A* and *B*. The cost of pumping can be obtained by summing the cost of pumping at site *A* plus site *B* ($Cost\ A + Cost\ B$); both of them depend on the water extracted Q_A and Q_B from sites *A* and *B*, respectively. Cost *A* is expressed by the equation:

$$Cost\ A = 8 + \frac{40 - 8}{17} * Q_A$$

Cost *B* is expressed by the equation:

$$Cost\ B = 15 + \frac{26 - 15}{13} * Q_B$$

For the well at site *A*, the water extracted (Q_A) must be equal to or greater than 0. The maximum sustainable groundwater extraction at site *A* is equal to or less than 17 acre-feet/year.

For the well at site *B*, the water extracted (Q_B) must be equal to or greater than 0. The maximum sustainable groundwater extraction at site *B* is equal to or less than 13 acre-feet /year.

The water required to satisfy the industry is 22 acre-feet/year.

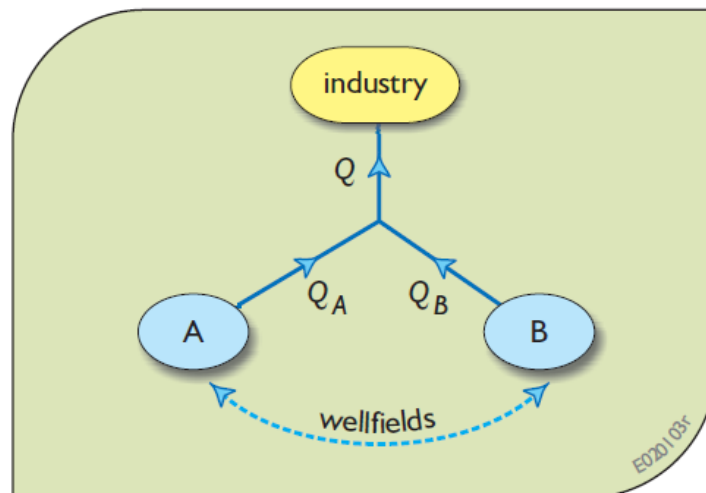


Figure 49

To be turned in:

Following the seven steps of linear programming (page 4 of this exercise)

- 1) Write down the *objective function* and the constraints
- 2) Draw the *feasible region*. You can use excel, or a simple piece of paper and scan it, or some engineering paper.
- 3) Obtain the vertices of the feasible region. Substitute the values of the vertices into the objective function. Submit a table showing the vertices of the feasible region as well as the

value of the objective function for each pair of Q_A and Q_B values (see slides 13 and 19 of the presentation of “Optimization”).

- 4) Based on the previous analysis, what is the minimum cost of pumping for this industry?
- 5) What is the optimal water extraction Q_A and Q_B for the minimum cost of pumping?

Now, let's use the Open Solver model of excel to solve this problem. Use the file Ex_5.xlsx - tab “4 Minimize” to create a linear optimization model. You can use as initial values for Q_A (cell G5) and Q_B (cell G6) 10 and 10 AF/year , respectively. Calculate *Cost A* in cell D8 using the formula provided above. Similarly, calculate *Cost B* in cell D11 using the formula provided above. Don't forget to link the objective function in cell D5 by summing the *Cost A* (Cell D8) plus *Cost B* (Cell D11). When creating the linear optimization model in Open Solver, don't forget to select “Minimize” instead of “Maximize” (Figure 17).

- 6) Turn in a screenshot of your linear model, similar to Figure 44.
- 7) What is the minimum cost of pumping (the optimum value of the objective function) obtained once the model has been run (cell D5)?
- 8) What are the optimal water extractions Q_A (cell G5) and Q_B (cell G6) for the minimum cost of pumping?
- 9) How do these results compare with the same analysis done before in questions 4 and 5 of this problem (see above)? Are these results similar?