# ESM 121

Water Science and Management

Exercise 6:

Simulation Modeling

and

Optimization Modeling



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# **Table of Contents**

Simulation Modeling	
River system	
Water Resources Performance Criteria	19
River and Reservoir System	
Optimization Modeling.	
Objective	
Steps for Linear Programming	
Exercise 1 (Adapted from McKinney)	
Exercise 2 (Adapted from Loucks and van Beek)	57
Exercise 3 The perfect outfit! Extra credits (+20%)	58
Exercise 4 (Adapted from Loucks and van Beek) Extra credits (+20%)	60

# **Simulation Modeling**

The aim of this exercise section is to explore basic concepts of operating and allocation policies. Using the water available for each user, you will define equations that allocate water for the river and among 3 users.

# **River system**

The water system is shown in Figure 1. There are 4 main water demands with different water requirements and priorities (Table 1).



Figure 1

This is description of the water allocation policy for the system shown in Figure 1.

- *River* has the first priority, at least 2 units of water should be left in the river (if available) before the rest of the users can take their water demand. Once the water demand of all the user have been supplied, if there is any water available, then the remaining water will be left in the river.
- User 3 has the second priority, and once the *River* has met its demand, then *User 3* will take as much water up to meet its water demand, which is 5 units.
- User 1 and User 2 share both the 3<sup>rd</sup> priority in the system. Once *River* and *User 3* have met their water demands, then *User 1* and *User 2* will take as much water as they can in

<u>the same proportion</u> up to their water demand is fulfilled, *User 1* = 2.5 units of water and *User 2* = 3 units of water.

To solve this exercise of water operation and allocation policy; first, you will estimate the allocation policy (Step 1), and second, you will *copy and paste* the formulas <u>OR</u> *lookup* this allocation policy when you are running the calculations (Step 2).



Figure 2. Step 1, water allocation policy and Step 2, mass balance model.

#### Defining the water allocation policies.

#### Water User: River

First, let's start working with *River*. According to the description provided by the allocation policy, water is left in the *River* using the following instructions:

*"River"* has the first priority, at least 2 units of water should be left in the river (if available) before the rest of the users can take their water demand. Once the water demand of all the user have been supplied, if there is any water available, then the remaining water will be left in the river."

This can be converted into equations as follow:

- 100

$$Q_{t}^{River} = \begin{cases} Q_{t}^{in} & if & Q_{t}^{in} < 2\\ Q_{t}^{in} - \sum_{i=1}^{1=3} X_{it} & if & Q_{t}^{in} > 2 + \sum_{i=1}^{1=3} X_{it} \\ 2 & if & Q_{t}^{in} > 2 \text{ and } Q_{t}^{in} < 2 + \sum_{i=1}^{1=3} X_{it} \end{cases}$$

- in

The first condition,  $Q_t^{River} = Q_t^{in}$ , refers to "*River* [..], at least 2 units of water should be left in the river (<u>if available</u>)", the second condition,  $Q_t^{River} = 2$ , refers to "*River* [..], at least <u>2 units of water</u> <u>should be left in the river</u> (if available)" and the third condition,  $Q_t^{River} = Q_t^{in} - \sum_{i=1}^{1=3} X_{it}$ , refers to "Once the water demand of all the user have been supplied  $[2 + \sum_{i=1}^{1=3} X_{it}]$ , if there is any water available, then the remaining water will be left in the river."

Before declaring these equations, it will be easier if we define names to the variables first. Go to cell C54. Then, go to the menu of Formulas and click on "Define Name". A dialogue window should come out, you should name the variable "River\_WD" as a short name of "River Water Demand". ". Very important, make sure that in "Scope:" drop down menu you select "**Part 2 - River**". By default the "Refers to:" section should be selecting the cell C54 where the cursor is located. If this is not showing, click on the "cells and arrow" icon on the right and select the cell

C54. By doing this, you are naming that cell "River\_WD" that you will be able to call it like that in excel, cool, isn't it!

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38	40.															
40	45.													OK		ancel
42																/

You will have to "Define Name" for the following variables and cells:

"Define Name"	Cell
Variable	
River_WD	C54
X1_WD	D54
X2_WD	E54
X3_WD	F54

If you had a problem naming your variables or not selecting the right cell, you can click on the Formulas/Name Manager where you can delete or edit you variables

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River_W	/D 2			='Part 2 -	River'!\$C	Workbo		
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X2_WD	3			='Part 2 -	River'!SES	Workbo		
3_WD	5			='Part 2 -	River'ISFS	Workbo		
								-
Refers to:								
$\times$ $\checkmark$ =	'Part 2 - River'	\$C\$54						1
							C	lose
10								

Now, lets go to Cell C6 and type the following equation:

The equation in Cell C6	The	equation	in	Cell	C6	is:
-------------------------	-----	----------	----	------	----	-----

"=IF(B6<=River\_WD,B6,IF(B6>River\_WD+SUM(X1\_WD,X2\_WD,X3\_WD),B6-

SUM(X1\_WD,X2\_WD,X3\_WD), River\_WD))"

H		ڻ ک	<b>&amp;</b> - 💉	친 💁	÷		Ex_3.xlsx -	Excel			
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SUM	1		$\land \checkmark f_x$	=IF(B6<=Rive	er_WD,B6,IF(B	5>River_WD+	SUM(X1_WD,X	2_WD,X3_WD), <mark>B6</mark> -SUI	M(X1_WD,X2_W	D,X3_WD), Ri	ver_WD))
	А	В	С	D	E IF(k	gical_test, [val	ue_if_true], [value	_if_false]) H	1	J	К
1											
2			Water	Allocation	(Units of W	ater)					
3			(Qt <sup>River</sup> )	(X <sub>1t</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )					
4			1	3	3	2					
5		Q <sub>t</sub> <sup>In</sup>	River	User 1	User 2	User 3			9.		
6		0.	=IF(B6<=Riv								
7		1.	1.						8.		Quiver
49											
50			Wa	ater Deman	ds						
51		Priority	1	3	3	2					
52		Name	River	User 1	User 2	User 3					
53		Nickname	(Qt <sup>River</sup> )	(X <sub>1t</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )					
54		Volume	2	2.5	3	5					
55											

# Figure 3

The first part of this equation, grey box, refers to the first condition, the green box refers to the third condition, and if none of the previous conditions applied, then it applies the result from condition 2.



Figure 4

Copy and paste this equation for the remaining of the column.

2	^	R	C	D	E		
1	A	D	C.	U	E	P	
2			Wate	or Demands	(Units of Wa	ter)	
2			(O River)	/v )	14 1		
2	_		(Ut )	(A <sub>1t</sub> )	(A2t)	(^3t)	
5		O In	Pivor	JIcor 1	JIcor 2	Licar 2	
5		0	0	USEL I	USEI 2	USEI 5	
7		1	1				
0		2	2				
0		2.	2				
10		3.	2				
11		4.	2				
12		4.5	2				
12		5.5	2				
14		6	2				
15		6.5	2				
16		7	2				
17		8	2				
18		0.	2				
19		10	2				
20		11	2				
21		12	2				
22		12.5	2				
23		13	2.5				
24		14	3.5				_
25		15	4.5			<b> </b>	
26		16.	5.5				
27		17.	6.5				
28		18.	7.5				
29		19.	8.5				
30		20.	9.5				
31		21.	10.5				
32		22.	11.5				
33		23.	12.5				
34		24.	13.5				
35		25.	14.5				
36		37.5	27				
37		38.	27.5				
38		40.	29.5				
39		42.	31.5				
40		45.	34.5				
41							

Figure 5

#### Water User: User 3

According to the description provided by the allocation policy, *User 3* has the second priority, and once the *River* has met its demand, then *User 3* will take as much water up to its water demand is fulfilled, which is 5 units.

This can be converted into equations as follow:

$$X_{3t} = \begin{cases} 0 & if & Q_t^{in} \le 2\\ Q_t^{in} - Q_t^{River} & if & Q_t^{in} > Q_t^{River} \text{ and } Q_t^{in} < Q_t^{River} + X_{3t}\\ 5 & if & Q_t^{in} \ge Q_t^{River} + X_{3t} \end{cases}$$

The first condition,  $X_{3t}=0$ , refers to "User 3 has the second priority, and <u>once the River has met its</u> <u>demand</u>, then User 3 [...];" the second condition,  $X_{3t}=5$ , refers to "User 3 [...] will take as much water <u>up to meet its water demand</u>, which is 5 units." and the third condition,  $X_{3t}=Q_t^{in}-Q_t^{River}$  refers to "User 3 [...] <u>will take as much water up to</u> its water demand is fulfilled, which is 5 units." The equation in Cell F6 is:

# "=IF(B6<=River\_WD,0,IF(AND(B6>River\_WD,B6<River\_WD+X3\_WD),B6-River WD,X3 WD))"

SUM			$\checkmark \checkmark f_x$	=IF(B6<=Rive	er_WD,0,IF(AN	ND(B6>River_V	ND,B6 <river_< th=""><th>WD+X3_WD)<mark>,B6-River_</mark>W</th><th>/D,X3_WD))</th></river_<>	WD+X3_WD) <mark>,B6-River_</mark> W	/D,X3_WD))
	А	В	С	D	E	F	G	Н	L
1									
2			Water	Allocation	(Units of W	/ater)			
3			(Qt <sup>River</sup> )	(X <sub>1t</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )			
4			1	3	3	2			
5		Q <sub>t</sub> <sup>In</sup>	River	User 1	User 2	User 3			9.
6		0.	0.			3_WD))			
7		1.	1.			0.	Ĩ		8.
49									
50			Wa	ater Deman	ds				
51		Priority	1	3	3	2			
52		Name	River	User 1	User 2	User 3			
53		Nickname	(Qt <sup>River</sup> )	(X <sub>1t</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )			
54		Volume	2	2.5	3	5			
55									

Figure 6

The first part of this equation, grey box, refers to the first condition, the green box refers to the third condition, and if none of the previous conditions applied, then it applies the result from condition 2.



Figure 7

Copy and paste this equation for the remaining of the column.

A	В	с	D	E	F
1					
2		Wate	er Demands	(Units of Wa	ater)
3		(Q, River)	(X <sub>1t</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )
4		1	3	3	2
5	Q <sub>t</sub> <sup>In</sup>	River	User 1	User 2	User 3
6	0.	0			0
7	1.	1			0
8	2.	2			0
9	3.	2			1
10	4.	2			2
11	4.5	2			2.5
12	5.	2			3
13	5.5	2			3.5
14	6.	2			4
15	6.5	2			4.5
16	7.	2			5
17	8.	2			5
18	9.	2			5
19	10.	2			5
20	11.	2			5
21	12.	2			5
22	12.5	2			5
23	13.	2.5			5
24	14.	3.5			5
25	15.	4.5			5
26	16.	5.5			5
27	17.	6.5			5
28	18.	7.5			5
29	19.	8.5			5
30	20.	9.5			5
31	21.	10.5			5
32	22.	11.5			5
33	23.	12.5			5
34	24.	13.5			5
35	25.	14.5			5
36	37.5	27			5
37	38.	27.5			5
38	40.	29.5			5
39	42.	31.5			5
40	45.	34.5			5
41					
40					

Figure 8

#### Water User: User 1 and 2

According to the description provided by the allocation policy, *User 1* and *User 2* share both the  $3^{rd}$  priority in the system. Once *River* and *User 3* have met their water demands, then *User 1* and *User 2* will take as much water as they can in the same proportion up to meet their water demand, *User 1 = 2.5* units of water and *User 2 = 3* units of water.

For User 1, this can be converted into equations as follow:

$$X_{1t} = \begin{cases} 0 & if \qquad Q_t^{in} \le Q_t^{River} + X_{3t} \\ \left(Q_t^{in} - \left(Q_t^{River} + X_{3t}\right)\right) \times \left(\frac{X_{2t}}{X_{1t} + X_{2t}}\right) & if \qquad Q_t^{in} - \left(Q_t^{River} + X_{3t}\right) < X_{1t} + X_{2t} \text{ and } Q_t^{in} > Q_t^{River} + X_{3t} \\ 2.5 & if \qquad Q_t^{in} - \left(Q_t^{River} + X_{3t}\right) > X_{1t} + X_{2t} \end{cases}$$

For User2, this description can be converted unto equations as follows:

$$X_{2t} = \begin{cases} 0 & if \qquad Q_t^{in} \le Q_t^{River} + X_{3t} \\ \left(Q_t^{in} - \left(Q_t^{River} + X_{3t}\right)\right) \times \left(\frac{X_{1t}}{X_{1t} + X_{2t}}\right) & if \qquad Q_t^{in} - \left(Q_t^{River} + X_{3t}\right) < X_{1t} + X_{2t} \text{ and } Q_t^{in} > Q_t^{River} + X_{3t} \\ \mathbf{3} & if \qquad Q_t^{in} - \left(Q_t^{River} + X_{3t}\right) > X_{1t} + X_{2t} \end{cases}$$

The first condition,  $X_{ll}=0$ , refers to "<u>Once *River* and *User 3* have met their water demands, then *User 1* and *User 2* will take [...];" the second condition,  $X_{lt}=2.5$ , refers to "[...] <u>User 1 and User</u> <u>2 will take as much water</u> as they can in the same proportion <u>up to meet their water demand</u>, *User*  1 = 2.5 units of water and *User 2* = 3 units of water" and the third condition, refers to "[...] <u>User</u> <u>1 and User 2</u> will take as much water as they can in the same proportion <u>up to</u> meet their water demand, *User 1* = 2.5 units of water and *User 2* = 3 units of water" The equation for *User 1* in Cell D6 is: "=IF(B6<=River\_WD+X3\_WD,0,IF(B6-</u>

(River\_WD+X3\_WD)>=X1\_WD+X2\_WD,X1\_WD,(B6-

(River\_WD+X3\_WD))\*(**X1\_WD**/(X1\_WD+X2\_WD))))"

× < fx =IF(B6<=River\_WD+X3\_WD,0,IF(B6-(River\_WD+X3\_WD)>=X1\_WD+X2\_WD,X1\_WD,(B6-(River\_WD+X3\_WD))\*(X1\_WD/(X1\_WD+X2\_WD))))

1					
2		Water	Allocation	(Units of W	ater)
3		(Qt <sup>River</sup> )	(X <sub>1t</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )
4		1	3	3	2
5	Q <sub>t</sub> <sup>In</sup>	River	User 1	User 2	User 3
6	0.	0.	_WD))))		0.
7	1.	1.	0.		0.
8	2.	2.	0.		0.
9	3.	2.	0.		1.

49					
50		Wa	ater Deman	ds	
51	Priority	1	3	3	2
52	Name	River	User 1	User 2	User 3
53	Nickname	(Qt <sup>River</sup> )	(X <sub>1t</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )
54	Volume	2	2.5	3	5
55					

#### Figure 9

The first part of this equation, grey box, refers to the first condition, the green box refers to the third condition, and if none of the previous conditions applied, then it applies the result from condition 2.



The similar equation for User to User 1, Cell E6 is: 2 is very in "=IF(B6<=River WD+X3 WD,0,IF(B6-

(River\_WD+X3\_WD)>=X1\_WD+X2\_WD,X2\_WD,(B6-

(River\_WD+X3\_WD))\*(**X2\_WD**/(X1\_WD+X2\_WD))))"

Copy and paste this equation for the remaining of the column.

	A B	С	D	E	F
1					
2		Wate	er Demands	(Units of Wa	ter)
3		(Qt River)	(X <sub>1t</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )
4		1	3	3	2
5	Q	River	User 1	User 2	User 3
6	0.	0.	0.	0.	0.
7	1.	1.	0.	0.	0.
8	2.	2.	0.	0.	0.
9	3.	2.	0.	0.	1.
10	4.	2.	0.	0.	2.
11	4.5	2.	0.	0.	2.5
12	5.	2.	0.	0.	3.
13	5.5	2.	0.	0.	3.5
14	6.	2.	0.	0.	4.
15	6.5	2.	0.	0.	4.5
16	7.	2.	0.	0.	5.
17	8.	2.	0.5	0.5	5.
18	9.	2.	0.9	1.1	5.
19	10.	2.	1.4	1.6	5.
20	11.	2.	1.8	2.2	5.
21	12.	2.	2.3	2.7	5.
22	12.5	2.	2.5	3.	5.
23	13.	2.5	2.5	3.	5.
24	14.	3.5	2.5	3.	5.
25	15.	4.5	2.5	3.	5.
26	16.	5.5	2.5	3.	5.
27	17.	6.5	2.5	3.	5.
28	18.	7.5	2.5	3.	5.
29	19.	8.5	2.5	3.	5.
30	20.	9.5	2.5	3.	5.
31	21.	10.5	2.5	3.	5.
32	22.	11.5	2.5	3.	5.
33	23.	12.5	2.5	3.	5.
34	24.	13.5	2.5	3.	5.
35	25.	14.5	2.5	3.	5.
36	37.5	27.	2.5	3.	5.
37	38.	27.5	2.5	3.	5.
38	40.	29.5	2.5	3.	5.
39	42.	31.5	2.5	3.	5.
40	45.	34.5	2.5	3.	5.
41					

Figure 10

The graph should look like:



Figure 11

# Linking the Water Allocation policy with the Mass Balance Model

There are two methods to use the allocation policy that was just derived into the *mass balance model*. Use **<u>EITHER</u>** of the following two methods, <u>**but only chose one**</u>, the one that you feel more comfortable using.

### **First Method**

The easiest way is to select cells C6 to F6 (C6:F6), copy (CTrl+C) and paste the formulas on cells into cells D45 to G45 (D45:G45).



Figure 12. On the left, Copy (Ctrl+C) cells C6 to F6, and paste them (Ctrl+V) into cells D45 to

#### G45

And then use these cells (D45:G45) to copy then through the rest of the table.

		E48	• (*	f <sub>x</sub>	=IF(C48<=\$0	\$41+\$F\$41	.,0,IF(C48-(	\$C\$41+\$F\$4
	А	В	С	D	E	F	G	Н
41		Volume	2	2.5	3	5		
42								
43				Mas	s Balance N	lodel		
44		Year	Q <sub>t</sub> <sup>in</sup>	(Qt <sup>River</sup> )	(X <sub>it</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )	Probation
45		2015	14	3.5	2.5	3.	5	14
46		2016	11	2	1.8	2.2	5	11
47		2017	9	2	0.9	1.1	5	9
48		2018	0	0	0.	0.	0	0
49		2019	7	2	0.	0.	5	7
50		2020	19	8.5	2.5	3.	5	19
51		2021	13	2.5	2.5	3.	5	13
52		2022	16	5.5	2.5	3.	5	16
53		2023	20	9.5	2.5	3.	5	20
54		2024	9	2	0.9	1.1	5	9
55		2025	15	4.5	2.5	3.	5	15
56		2026	7	2	0.	0.	5	7
57		2027	16	5.5	2.5	3.	5	16
58		2028	12	2	2.3	2.7	5	12
59		2029	26	15.5	2.5	3.	5	26
60		2030	21	10.5	2.5	3.	5	21
61		2031	23	12.5	2.5	3.	5	23
62		2032	7	2	0.	0.	5	7
63		2033	5	2	0.	0.	3	5
64		2034	11	2	1.8	2.2	5	11
65		2035	8	2	0.5	0.5	5	8
66								

Figure 13

#### Second Method

The second way to do this, is a little bit more elegant, but still the same principle. This method is used, and it is very helpful, when managing large amount of water users. In this case, what you want is to have control of the water allocating policy in one table (B6:F40) and to recall values from this table in the mass balance model (B59:H79). You can create a "lookup" command that will find the values estimated in the allocation policy table. In cell D59, let's insert a command that will look up for the value of  $Q_t^{River}$  in the table water allocation table using the following equation:

#### "=VLOOKUP(\$C59,\$B\$6:\$F\$40,2,TRUE)"

What the previous command means, is that excel will <u>lookup vertically</u> ("VLookup") for the  $Q_t^{in}$  value of cell C59 (in this case 14) in the first column (it always start looking in the first column) of the table specified by the cell coordinates "\$B\$6:\$F\$40" (the green box in the following figure),

and it will retrieve the value of column number 2 of the specified table, which is  $Q_t^{River}$ , in this case for 14 is 3.5. When using this command, make sure to specify the column number that you want values to retrieve, and excel will <u>always</u> use the first column as the column to lookup for numbers, in this case, it will look for the "approximate" value in cell C58 on the first column of the table specified, that's why at the end of the command you need to write "TRUE".



Figure 14

For User 1,  $X_{lt}$ , following equation cell E59: use the in "=VLOOKUP(\$C59,\$B\$6:\$F\$40,**3**,TRUE)" User 2, following For  $X_{2t}$ , the equation in cell F59: use "=VLOOKUP(\$C59,\$B\$6:\$F\$40,4,TRUE)"

# For User 3, X<sub>3t</sub>, use the following equation in cell G59: "=VLOOKUP(\$C59,\$B\$6:\$F\$40,**5**,TRUE)"

Results from the Mass Balance model should look like the following pictures

								-			
		E48	• (**	f <sub>x</sub>	=IF(C48<=\$C\$41+\$F\$41,0,IF(C48-(\$C\$41+\$F\$41)						
	Α	В	с	D	E	F	G	Н			
41		Volume	2	2.5	3	5	_				
42											
43				Mas	s Balance IV	Iodel					
44		Year	Qtin	(Q <sup>River</sup> )	(X <sub>it</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )	Probation			
45		2015	14	3.5	2.5	3.	5	14			
46		2016	11	2	1.8	2.2	5	11			
47		2017	9	2	0.9	1.1	5	9			
18		2018	0	0	0.	0.	0	0			
49		2019	7	2	0.	0.	5	7			
50		2020	19	8.5	2.5	3.	5	19			
51		2021	13	2.5	2.5	3.	5	13			
52		2022	16	5.5	2.5	3.	5	16			
53		2023	20	9.5	2.5	3.	5	20			
54		2024	9	2	0.9	1.1	5	9			
55		2025	15	4.5	2.5	3.	5	15			
56		2026	7	2	0.	0.	5	7			
57		2027	16	5.5	2.5	3.	5	16			
58		2028	12	2	2.3	2.7	5	12			
59		2029	26	15.5	2.5	3.	5	26			
60		2030	21	10.5	2.5	3.	5	21			
61		2031	23	12.5	2.5	3.	5	23			
62		2032	7	2	0.	0.	5	7			
63		2033	5	2	0.	0.	3	5			
64		2034	11	2	1.8	2.2	5	11			
65		2035	8	2	0.5	0.5	5	8			
66											





Figure 16

### To be turned in:

1) Show a table like the one calculated in Figure 15 and the series of graphs calculated in Figure 16.

# Water Resources Performance Criteria

Phew, we have done a lot of work to describe the water allocation system for four water demands (River, X1, X2, and X3), imagine in a basin where we have hundreds and sometimes thousands of users! All the previous work quantitatively describes how water is allocated, but it does not gives us any performance of the water supply. For estimating the water supply performance, there are four water resources performance criteria used widely in the field: Reliability (in time and volume), Vulnerability, and Resilience. I will show you how to calculate those water resources performance criteria to evaluate the our water resources system.

#### **Reliability (Time and Volume)**

Have you ever heard in the news about "Water Supply Reliability"? do you remember Governor Brown in our last drought when he talked about "making sure everyone had a reliable water supply during the drought? Water Supply <u>Reliability</u> is a common term used to quantify how frequent a water demand is met (time-based reliability), and how close to the total amount demanded (volumetric reliability). The *time based reliability* is the portion of time that water demand is fully supplied, or in other words, <u>how frequent a water demand is fully supplied</u>.

To estimate the time based reliability, we need to estimate *water supply deficits*, which is the difference between the water demand minus the water supplied:

$$Deficit_{t}^{i} = \begin{cases} X_{Demand,t}^{i} - X_{Supplied,t}^{i} & if X_{Demand,t}^{i} > X_{Supplied,t}^{i} \\ 0 & if X_{Demand}^{i} = X_{Supplied,t}^{i} \end{cases}$$

where:  $X^{i}_{Demand,t}$  is the water demand for the  $i^{\text{th}}$  water user, and  $X^{i}_{Supplied,t}$  is the water supplied in the  $t^{\text{th}}$  time period. Finally, the reliability for the  $i^{\text{th}}$  user is:

$$Reliability(time)^{i} = \frac{\# of times Deficit_{t}^{i} = 0}{n}$$

where n is the total number of time steps, often used the total number of years or months, depending on the water management unit of time.

The volumetric reliability is the sum of the deficits divided by sum of the water demand. <u>The</u> volumetric reliability express how much water was supplied compared with the total amount of water demanded.

$$Reliability(volume)^{i} = \frac{\sum X_{Supplied,t}^{i}}{\sum X_{Demand,t}^{i}}$$

First, let's calculate the matrix of Water Supply Deficits. As mentioned before, a water supply deficit is the difference between the water demand minus the water supplied. For the River demand, a deficit only exist if the water supplied is less than the water demand  $(Q_t^{River} = 2 \text{ units of } d_t)$ cell Let's estimate the deficit for year 2020 in D87 follows: water). as "=IF(D59<River WD,River WD-D59,0)"

SUM	и	• :	× v	$f_X$	=IF(D59<	River_WI	D,River_	WD-D59,0)
4	A	В	с	D	E	F	G	н
51		Priority	1	3	3	2		
52		Name	River	User 1	User 2	User 3		
53		Nickname	(QtRiver)	(X <sub>11</sub> )	(X <sub>21</sub> )	(X <sub>37</sub> )		
54		Volume	2	2.5	3	5		
55								
56								
57				N	lass Balanc	e Model		
58		Year	Q, <sup>in</sup>	(Q <sup>River</sup> )	(X <sub>21</sub> )	(X <sub>21</sub> )	(X <sub>31</sub> )	Double Checking
59		2020	14	3.5	2.5	3.	5.	14
60		2021	11	2.	1.8	2.2	5.	11
61		2022	9	2.	0.9	1.1	5.	9
62		2023	0	0.	0.	0.	0.	0
63		2024	9	2.	0.9	1.1	5.	9
64		2025	19	8.5	2.5	3.	5.	19
65		2026	13	2.5	2.5	3.	5.	13
66		2027	16	5.5	2.5	3.	5.	16
67		2028	20	9.5	2.5	3.	5.	20
68		2029	9	2.	0.9	1.1	5.	9
69		2030	15	4.5	2.5	3.	5.	15
70		2031	7	2.	0.	0.	5.	7
71		2032	16	5.5	2.5	3.	5.	16
72		2033	12	2.	2.3	2.7	5.	12
73		2034	26	15.5	2.5	3.	5.	26
74		2035	21	10.5	2.5	3.	5.	21
75		2036	23	12.5	2.5	3.	5.	23
76		2037	7	2.	0.	0.	5.	7
77		2038	5	2.	0.	0.	3.	5
78		2039	11	2.	1.8	2.2	5.	11
79		2040	8	2.	0.5	0.5	5.	8
80								
81								
82								
83								
84								
85				V	Vater Supp	oly Deficits		
86			Year	(Qe <sup>River</sup> )	(X1.)	(X <sub>2t</sub> )	(X <sub>3t</sub> )	
87			2020	=IF(D59 <r< td=""><td></td><td></td><td></td><td></td></r<>				
88			2021					
89			2022					

If you copy and paste the equation of cell D87 in cells D88:D107, you will realize that only one year (2023) the river was not able to meet it's water demand of 2 units of water, with a deficit of two units.



You can count the number of time steps (n), the number of deficits (# of deficits) and the total volume of deficits (Sum of deficits) as follows:

n -> Cell D109: "=COUNT(D87:D107)"

# of deficits -> Cell D110: "=COUNTIF(D87:D107,">0")"

# of NO deficits -> Cell D111: "=COUNTIF(D87:D107,"=0")"

Sum of deficits -> Cell D112: "=SUMIF(D87:D107,">0")"

Sum of Water Demand -> Cell D113: "=River\_WD\*D109"

Sum of Water Supplied -> Cell D114: "=D113-D112"

		Water Supply Deficits						
	Year	(Q <sub>t</sub> <sup>River</sup> )	(X <sub>1t</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )			
	2020	0						
	2021	0						
	2022	0						
	2023	2						
	2024	0						
	2025	0						
	2026	0						
	2027	0						
	2028	0						
	2029	0						
	2030	0						
	2031	0						
	2032	0						
	2033	0						
	2034	0						
	2035	0						
	2036	0						
	2037	0						
	2038	0						
	2039	0						
	2040	0						
	n =	21						
# 0	of deficits =	1						
# of N	0 deficits =	20						
Sum o	of deficits =	2						
Sum of the Water	r Demand =	42						
Sum of the Water	Supplied =	40						

According to the equation of reliability in time, we can estimate it as follows:

$$Reliability(time)^{i} = \frac{\# of times Deficit_{t}^{i} = 0}{n}$$

Reliability in time -> Cell D116: "=D111/D109". The time-based reliability means that 95% of the time the water demand of the river can be fully supplied.

SUN	∧ <b>-</b> :	(D109			
	А	В	С	D	Е
108					
109			n =	21	
110		#	of deficits =	1	
111		# of N	IO deficits =	20	
112		Sum	of deficits =	2	
113	Sum	of the Wate	r Demand =	42	
114	Sum	of the Wate	r Supplied =	40	
115					
116		Reliabi	ility (time) =	=D111/D1	
117		Reliability	/ (volume) =	95%	
118		Vu	Inerability =		
119			Resilience =		
100					

According to the equation of volumetric reliability, we can estimate it as follows:

$$Reliability(volume)^{i} = \frac{\sum X_{Supplied,t}^{i}}{\sum X_{Demand t}^{i}}$$

 $\Delta \Delta_{Demand,t}$ Volumetric Reliability -> Cell D117: "=D114/D113". The volumetric reliability means that 95%

of the volume requested by the River was able to be supplied.

SUN	∧ <b>→</b> :	D113			
	А	В	С	D	E
108					
109			n =	21	
110		#	of deficits =	1	
111		# of N	IO deficits =	20	
112		Sum	of deficits =	2	
113	Sum	of the Wate	r Demand =	42	
114	Sum	of the Wate	r Supplied =	40	
115					
116		Reliabi	lity (time) =	95%	
117		Reliability	(volume) =	4/D113	
118		Vu	Inerability =		
119			Resilience =		
100					

# Vulnerability

The performance criterion of vulnerability expresses the severity of the deficits. In summary, it is the average of the water supply deficits, expressed as a percentage of the water demand. Vulnerability can be calculated as follows:

$$Vulnerability^{i} = \frac{\left(\frac{\sum Deficit_{t}^{i}}{\# of \ times \ Deficit_{t}^{i} > 0 \ occured}\right)}{Water \ Demand^{i}}$$

Vulnerability -> Cell D118: "=(D112/D110)/River\_WD". The vulnerability criterion means that when a deficit happens, on average, the magnitude of the deficit is 100% of the water demanded by the river.

SUM	-	· : >	<	<i>f</i> <sub>x</sub> =(0	0112/D110	)/River_	WD
	А	В	с	D	E	F	G
108							
109			n =	21			
110		# 0	of deficits =	1			
111		# of N	0 deficits =	20			
112		Sum o	of deficits =	2			
113	Sum o	of the Wate	r Demand =	42			
114	Sum o	f the Water	Supplied =	40			
115							
116		Reliabi	lity (time) =	95%			
117		Reliability	(volume) =	95%			
118		Vul	nerability =	=(D112/D1			
119		Re	esilience =				
120							
121							

One tricky thing about the Vulnerability is to calculate it when no deficit occur. For this case, the system is considered 0% vulnerable. Thus, the equation in Cell D118 should be changed to account for this peculiar condition as follows: "= $IF(D110=0,0,(D112/D110)/River_WD)$ ".

SUN	SUM • : × • fx =IF(D110=0,0,(D112/D110)/River_WD)											
	А	В	С	D	E	F						
108												
109			n =	21								
110		#	of deficits =	1								
111		# of N	NO deficits =	20								
112		Sum	of deficits =	2								
113	Sum	of the Wate	er Demand =	42								
114	Sum	of the Wate	r Supplied =	40								
115												
116		Reliab	ility (time) =	95%								
117		Reliability	<mark>/ (volume)</mark> =	95%								
118		Vu	Inerability =	'er_WD)								
119			Resilience =	100%								
120		Sustainab	ility Index =	98%								
121												

## Resilience

Resilience is the system's capacity to adapt to changing conditions, resilience must be considered as a statistic that assesses the flexibility of water management policies to adapt to changing conditions. The classic definition of resilience is the probability that a system recovers from a period of failure, e.g., a deficit in water supply. For our specific case, <u>Resilience is the probability that a year of no-deficit follows a year of deficit</u> in the water supply for the *i*<sup>th</sup> water user. Resilience is a useful statistic to assess the recovery of the system once it has failed. Resilience is expressed as:

$$Resilience^{i} = \frac{\# of \ times \ Deficit_{t}^{i} = 0 \ follows \ Deficit_{t}^{i} > 0}{\# of \ times \ Deficit_{t}^{i} > 0 \ ocurred}$$

To estimate Resilience, we need to create a matrix that track the number of times a period of No Deficit follow a period with Deficit. We will always start the first time step with zeros.



For year 2021, we can estimate if the previous year was in a water supply deficit AND this year is not in a deficit in cell I88 as follows: "=IF(AND(D88=0,D87>0),1,0)". This equation flags (inserting a 1) if the previous year was in a deficit and the current year is not. The following figure

shows that only in year 2024 the previous condition is true, meaning that the previous year (2023) was in a water supply deficit and that the current year (2024) it is not.

	1	water Supp	ply Deficits	5		Resil	ience	
Year	(Q, <sup>River</sup> )	(X <sub>1k</sub> )	(X <sub>21</sub> )	(X <sub>34</sub> )	(Q, <sup>River</sup> )	(X <sub>1k</sub> )	(X <sub>24</sub> )	(X <sub>31</sub> )
2020	0				0	0	0	0
2021	0				0			
2022	0				0			
2023	2				0			
2024	0				1			
2025	0				0			
2026	0				0			
2027	0				0			
2028	0				0			
2029	0				0			
2030	0				0			
2031	0				0			
2032	0				0			
2033	0				0			
2034	0				0			
2035	0				0			
2036	0				0			
2037	0				0			
2038	0				0			
2039	0				0			
2040	0				0			

You can sum all of the times this condition occurs in Cell I109: "=SUM(I87:I107)"

The resilience can be calculated as follows:

$$Res^{i} = \frac{\# of times \ Deficit_{t}^{i} = 0 \ follows \ Deficit_{t}^{i} > 0}{\# of times \ Deficit_{t}^{i} > 0 \ ocurred}$$

Resilience -> Cell D119: "=IF(I109=0,1,I109/D110)". The resilience criterion means the probability that a water demands recovers from a deficit, when they occur. For the River water demand, there is a 100% probability that the system will recover.

		vater Sup	ply Deficits	5			Resi	ience	
Ye	ar (Q, <sup>River</sup> )	(X <sub>18</sub> )	(X21)	(X <sub>31</sub> )		(Q, River)	(X <sub>11</sub> )	(X <sub>28</sub> )	(X <sub>31</sub>
20	20 0					0	0	0	0
20	21 0					0			
20	22 0					0			
20	23 2					0			
20	24 0					1			
20	25 0					0			
20	26 0					0			
20	27 0					0			
20	28 0					0			
20	29 0					0			
20	30 0					0			
20	31 0					0			
20	32 0					0			
20	33 0					0			
20	34 0					0			
20	35 0					0			
20	36 0					0			
20	37 0					0			
20	38 0					0			
20	39 0					0			
20	40 0					0			
	n= 21				Count Besiliences	1			
# of defi	oits = 1								
# of NO defi	aits = 20								
Sum of defi	oits = 2								
Sum of the Water Dema	nd = 42								
oum of the Water Suppl	ied = 40								
Beliability (tir	nels 95%								
Beliability (un	nel= 95%								
Vulperab	litu = 100%								
Resilien	ce = 100%								

One tricky thing about the Resilience is to calculate the resilience of a system when no deficit occurs. For this case, the system is considered 100% resilient. Thus, the equation in Cell D119 should be changed to account for this peculiar condition as follows: "=IF(I109=0,1,I109/D110)".

SUM	· ·	× 🗸	<i>f</i> <sub>x</sub> =IF(110	)9=0,1,I109/D	110)					
	А	В	С	D	Е	F	G	Н	1	
108										
109			n =	21				Count Resilience=	1	1
110		#	of deficits =	1						
111		# of N	NO deficits =	20						
112		Sum	of deficits =	2						
113	Sum	of the Wate	er Demand =	42						
114	Sum	of the Wate	r Supplied =	40						
115										
116		Reliab	ility (time) =	95%						
117		Reliability	y (volume) =	95%						
118		Vu	Inerability =	100%						
119			Resilience =	=IF(I109=0						
120		Sustainab	ility Index =							
121										

# Water Resources Sustainability Index

Now, think about this: How do you know that a given user is receiving adequately their water demand? So far, we have estimated four performance criteria, but imagine that we start evaluating several alternative water management strategies and all of a sudden some criteria starts going up other going down. Can we estimate a single number (an Index) that summarize the performance

of a given water demand? In order to summarize all the performance criteria, we used index that helps synthesizing results. The Water Resources Sustainability Index (SI) is an index that aggregates a set of performance criteria, it is the geometric average of a group of m performance criteria for a given water user i:

$$SI^{i} = \left[\prod_{m=1}^{M} \text{Performance Criteria}_{m}^{i}\right]^{1/M}$$

For our case, the SI can be expressed as:

$$SI^{i} = \left[Rel(time)^{i} * Rel(Volume)^{i} * Res^{i} * (1 - Vuln^{i})\right]^{1/4}$$

For the River water demand, the SI can be estimated in Cell D120 as follows: Cell D120: "=(D116\*D117\*D119\*(1-D118))^(1/4)"

SUM	SUM • : × ✓ f <sub>x</sub> =(D116*D117*D119*(1-D118))^(1/4)											
	А	В	с	D	E	F	G	н				
108												
109			n =	21				Count Resilience=				
110		# 0	of deficits =	1								
111		# of N	0 deficits =	20								
112		Sum o	of deficits =	2								
113	Sum (	of the Wate	r Demand =	42								
114	Sum o	of the Water	Supplied =	40								
115												
116		Reliabi	lity (time) =	95%								
117		Reliability	(volume) =	95%	I							
118		Vul	nerability =	100%	I							
119		R	esilience =	100%								
120		Sustainabi	lity Index =	))^(1/4)	l l							
121												

Figure 17. Results of the Performance Criteria and Sustainability Index for the River Water

demand

For the River water demand, the SI = 0% meaning that is unsustainable. While the Reliability (in time and volume) is very high (95%) and the resilience is very high (100%), when a deficit occurs, this deficit is very damaging, 100% of the water demand. That's the reason why the SI = 0%

# Instructions

Calculate the deficit matrix for users 1, 2 and 3.

<u>Hint</u>: for year 2020, the deficit can be calculated as follows:

User1 -> Cell E87: "=IF(E59<X1\_WD,X1\_WD-E59,0)"

User2 -> Cell F87: "=IF(F59<X2 WD,X2 WD-F59,0)"

User3 -> Cell G87: "=IF(G59<X3\_WD,X3\_WD-G59,0)"

Calculate the following variables for Users 1, 2 and 3: n, # of deficits, # of NO deficits, Sum of

deficits, Sum of Water Demand, and Sum of Water Supplied

Hint: Remember to use the appropriate water demands for user 1 (X1\_WD), 2 (X2\_WD),

and 3 (X3\_WD) in the equation of Sum of Water Demand.

Calculate the resilience matrix for Users 1, 2 and 3

Estimate the Reliability (time-based and volumetric), Vulnerability, resilience and Water resources

Sustainability Index for user 1, 2 and 3.

Hint: Remember to use the appropriate water demands for user 1 (X1\_WD), 2 (X2\_WD),

and 3 (**X3\_WD**) in the equation of Vulnerability.

#### To be turned in:

1) Show table (like figure 17) of the four performance criteria [Reliability(time and volume), vulnerability and resilience) and the Sustainability Index.

- 2) Change the value of the *River* water demand  $(Q_t^{River})$  to 4 units in cell C54. Show a table like the one calculated in Figure 15 and the series of graphs calculated in Figure 16.
- 3) Show table (like figure 17) of the four performance criteria [Reliability(time and volume), vulnerability and resilience) and the Sustainability Index for all water demands for  $Q_t^{River} = 4$  units. Does the water supply for *Users 1, 2 and 3* change? Does it improved or got worse?
- 4) Calculate the average annual flow coming from upstream  $(Q_t^{In})$  (Average of cells C59:C79). How much the River water demand represent in comparison to the average annual inflows [River\_WD / Avg.  $(Q_t^{In})$ ]? Do you think this is enough water so the ecosystem can be sustained? Imagine that from your total monthly income (let say \$2,000), this monthly stipend is reduced to the same percentage of water that was left in the river (let say 30% of \$2,000 = \$600). Can you adequately live with this budget in a month? Do you see any similitude with what we have left to the river? Elaborate why there may be a conflict with human and environmental water supply.

# **River and Reservoir System**

Now, you are going to be working with a reservoir system, how exciting!

Before declaring any equation, it will be easier if we define names in this workbook as you did in the previous part of the exercise. Once again, make sure you are in the "**Part 2 – Reservoir**" sheet. Go to cell C54. Then, go to the menu of Formulas and click on "Define Name". A dialogue window should come out, you should name the variable "River\_WD" as you did before which as a short name of "River Water Demand". Very important, make sure that in "Scope:" drop down menu you select "**Part 2 - Reservoir**". By default the "Refers to:" section should be selecting the cell C54 where the cursor is located. If this is not showing, click on the "cells and arrow" icon on the right and select the cell C54. By doing this, you are naming that cell "River\_WD" that you will be able to call it like this worksheet in excel, cool, isn't it!

	5	ð	<b>2</b> -	*	\$⊃	_ <mark>⊘</mark> •			Ex 3	l.xlsx - Exce	el						
File	Hom	ie	- Insert	Page	Layout	Formulas	Data	Review	View	Develo	per A	crobat	<b>Ω</b> Te	New Name	2	?	$\times$
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31 32 33 34 35 26	21. 22. 23. 24. 25.													Defension			~
37 38 39 40 41 42	38. 40. 42. 45.													Kerers to:	='Part 2 - River'!\$C\$54 OK	C	ancel

You will have to "Define Name" for the following variables and cells:

"Define Name"	Cell
Variable	
River_WD	C54
X1_WD	D54

X2_WD	E54
X3_WD	F54
K	J54
Hedging	K54

If you had a problem naming your variables or not selecting the right cell, you can click on the

Formulas/Name Manager where you can delete or edit you variables

H	<b>५</b> ∂	۰ 2	*	₽	<u> </u>			Ex_3
File	Home	Insert	Page	Layout	Formulas	Data	Review	View
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	Part 2 - River'!	\$C\$54					C	llose

The same file has a system set up with a reservoir, with capacity K=25 units, and initial storage

 $S_0 = 7$  units, as shown below.

	Wate	r Demand	S	
Priority	1	3	3	2
Name	River	User 1	User 2	User 3
Nickname	(Qt <sup>River</sup> )	(X <sub>1t</sub> )	(X <sub>2t</sub> )	(X <sub>3t</sub> )
Volume	2	2.5	3	5



Figure 17

The only difference in the allocation policy is that for *User 1*, *User 2* and *User 3*, you will have to substitute  $S_{t-1}+Q_t^{in}$  instead of  $Q_t^{in}$  in each of the conditionals. For River,  $Q_t^{River}$ , the water allocation changes as follow:

$$Q_t^{River} = \begin{cases} Q_t^{in} + S_{t-1} & if \quad Q_t^{in} + S_{t-1} < 2 \\ 2 & if \quad Q_t^{in} + S_{t-1} \geq 2 \end{cases}$$

In addition, reservoir spills must be calculated (Column G, Cells G6:G40) as follow:

$$Q_t^{Spill} = \begin{cases} 0 & \text{if } S_{t-1} + Q_t^{in} \le K + Q_t^{River} + \sum_{i=1}^{i=3} X_{it} \\ \left(S_{t-1} + Q_t^{in}\right) - \left(K + Q_t^{River} + \sum_{i=1}^{i=3} X_{it}\right) & \text{if } S_{t-1} + Q_t^{in} > K + Q_t^{River} + \sum_{i=1}^{i=3} X_{it} \end{cases}$$

Please write down a paragraph explaining in plain words how the operation for River has changed, such as the one used in the River System:
 *"River* has the first priority, at least 2 units of water should be left in the river (if available) before the rest of the users can take their water demand. [Insert here how the water allocation policy has changed for River]."

Hint: In order for the river to receive extra water, does the reservoir have to spill?

- 2) Calculate the spills (Column G, Cells G6:G40) using the equation provided above.
- 3) Estimate the table for the water allocation policy (cells B2:H40) and show this table.
- 4) Estimate in column D the  $S_{t-1}+Q_t^{in}$ .
- 5) Use any of the two methods explained above to link the water allocation policy table with the mass balance model for the River (E59:E79), User 1 (F59:F79), User 2 (G59:G79) and User 3 (H59:H79).
- Using the mass balance equation, estimate the reservoir storage through every time step (I59:I79). Remember that the storage can't exceed the storage capacity K=25 units.
- 7) Use any of the two methods explained above to link the water allocation policy table with the mass balance model for Reservoir spills (J59:J79),
- 8) Display the mass balance results, like figure 15 and 16.
- 9) Similarly, does the water supply for Users 1, 2 and 3 improved if the reservoir is built? Show a comparison of the water supply for both systems (With and without a reservoir  $Q_t^{River} = 2$  units in both calculations)
- 10) What about the River? Does the river improve their water supply?
- 11) Show table (like figure 17) of the four performance criteria [Reliability(time and volume), vulnerability and resilience) and the Sustainability Index.
- 12) Did the performance of the systems improved or got worse? Create a table showing a comparison the performance criteria and

# **Optimization Modeling**

# Objective

The objective of this exercise is to provide a set of problems using simple optimization techniques as well as the basics of linear programming using a linear optimization solver in Excel.

# **Useful Formulas**

Find the decision variables, x, that optimizes (maximizes or minimizes) an objective function.

For instance, to minimize:





# Steps for Linear Programming

In order to solve linear programing, these are 7 steps that you should follow to identify the maximum or minimum value for the objective function at hand:

- 1. Define the *optimization purpose*. Is the objective to **Maximize or Minimize**?
- Define Objective Function, or in other words, write the Objective Function into an equation. If the equation is linear (Z=ax1+bx2), it can be solved through a linear programming. If not, other techniques of non-linear programming can be used to solve this type of problem.
- Define the Constraints. Write the constraints into inequalities, so they can be used to define the Feasible Region. Notice that the statement "the value of x is greater than (or at least) 20" means x≥20, and the statement "the value of x is smaller than (or less than) 20" means x≤20
- 4. Define the Feasible Region. Use the constraints (inequalities) to bound the feasible region. For constraints with the form "ax<sub>1</sub>+bx<sub>2</sub>≤C", convert them into equations by dropping the "≤" or "≥" symbol and adding the equal sign "=", such as "ax<sub>1</sub>+bx<sub>2</sub>=C". Then solve the equation for x<sub>1</sub>=0 to obtain the point where the linear equation crosses the x<sub>1</sub> axis. Similarly, solve for x<sub>2</sub>=0 to obtain the point where the linear equation cross the x<sub>2</sub> axis.
- 5. Obtain the vertices of the feasible region. Do this by identifying the points in the feasible region and, for the constraints with the form "ax1+bx2≤C", convert them into equations "ax1+bx2=C" and obtain the value of the unknown variable, either x1 or x2, by solving the equation.
- Substitute vertices into the Objective Function. Substitute the values of the variables (x1 and x2) at the vertices into the objective function equation.
- 7. Select the values of the variables that are the maximum or minimum for the objective function, depending on the definition of the objective function (Step 1).

# **Installing Open Solver - Model**

Installing the Open Solver in excel.

1.- Copy the OpenSolver21.zip into your computer.

## 2.- Unzip this file

Name Name	Date modified	Туре	Size
📰 cbc	2/24/2012 8:45 PM	Application	2,913 KB
CPL License	7/7/2011 12:35 PM	Text Document	12 KB
GNU GPL License	7/7/2011 12:35 PM	Text Document	35 KB
📳 OpenSolver ChangeLog	9/5/2012 2:45 PM	Microsoft Excel Worksheet	18 KB
📳 OpenSolver	9/6/2012 10:06 PM	Microsoft Excel Add-In	467 KB
ReadMe	11/12/2011 6:55 AM	Text Document	5 KB

3.- Double click on the file: "OpenSolver" which is the add-In

Name -	Date modified	Туре	Size
🔲 cbc	2/24/2012 8:45 PM	Application	2,913 KB
CPL License	7/7/2011 12:35 PM	Text Document	12 KB
GNU GPL License	7/7/2011 12:35 PM	Text Document	35 KB
OpenSolver ChangeLog	9/5/2012 2:45 PM	Microsoft Excel Worksheet	18 KB
🚰 OpenSolver	9/6/2012 10:06 PM	Microsoft Excel Add-In	467 KB
ReadMe	11/12/2011 6:55 AM	Text Document	5 KB

4.- If a dialogue-window pops up, please select "enable macros"



5.- On the data menu of Excel you should see the Open Solver add-in in the right corner of the menu.

<b>X</b>	Microsoft Exc.	1	
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Get External Data Connectio	Sort & Filter D	ata Tools Outline	G Analysis OpenSolver
▼ (* f <sub>x</sub> )			Y

6.- Finally Open the file Ex\_5.xlsx

**Important note**.- If you can't open the solver, you can use the built-in solver that excel already has. Follow the instructions of this <u>video</u> to enable the excel solver.

# Exercise 1 (Adapted from McKinney).

Based on an annual water allocation of 1,800 acre-feet (AF), an irrigation district wants to know how they can maximize their profits by growing two types of crops, Crop A and Crop B. The profits can be obtained by summing the profit ( $C_A$  and  $C_B$ ) times the acreage ( $X_A$  and  $X_B$ ) for each crop. The irrigation district has constraints. The first constraint is the water use, which is the sum of the water requirement times the acreage for each crop, which must be equal or less than the water allocation (1,800 AF). Second, there are limits to growing one crop or another. The maximum acreage on which to grow crop A is 400 acre, and crop B is 600 acres. Lastly, the acreage must be a positive number for both crops. Table 1 shows the water requirement, expected profit per acre and maximum area cultivated for each crop.

	Crop A	Crop B
Water requirement (Acre feet/acre)	3	2
Profit (\$/acre)	300	500
Max area (acres)	400	600

T-1.1. 1

#### To be turned in:

- 1) Write down the objective equations and constraints for this problem. Take a look at the "Optimization Presentation", this exercise was explained in this presentation.
- 2) Copy the chart for the feasible region located in Ex 5.xlsx file tab "1 Maximize"
- 3) What was the solution (Maximum value for the objective function Z) in the presentation?

Now, let's work on the Ex\_5.xlsx file. Open the tab "1 Maximize". The right side of that tab should look like Figure 1. Every cell that is orange means you must declare some information there.

#### Variables and Objective Function

First, let's start with the *Variables*. We need to give the model a first guess of the variable, usually any value inside the feasible region and the model will take care to later give us the right result once we have run it. For  $X_A$  lets define an initial value of 400 (cell T4=400), and for  $X_B$  a value of 600 (cell T5=600), see Figure 2.





Now let's define the Objective function in cell Q5 as the sum of the multiplication of the acreage times the profits for each crop (cell Q5 =T4\*W4+T5\*W5), see Figure 3.

	TREND	) -	• (• 🗙 🖌 f <sub>*</sub>	=T4*W4+T	5*W5					
	0	Р	Q	R	S	Т	U	V	W	Х
1	Objective	Function								
2	Maximize	Z			Variables			Constants		
3	Z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Acreage			Profit		
4	Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres	C <sub>A</sub> =	\$300	/acre
5	Z	=	=T4*W4+T5*W5		X <sub>B</sub> =	600	acres	C <sub>B</sub> =	\$500	/acre
-							T	1		



# Constraints

Now let's work on the *Constraints*. Constraint 1 specifies that the acreage of crop a ( $X_A$ ) must be equal to or less than 400 acres. Link cell O10 with the value of XA (cell T4) by typing in cell O10 "=T4"; see Figure 4. Type in cell Q10 the constraint value of 400 (Figure 5).

	TREND	) <del>•</del> (	🗏 X 🗸 :	<i>f</i> <sub>x</sub> =T4					
	М	N	0	Р	Q	R	S	т	U
1			Objective	Function					
2	ctive		Maximize	Z			Variables		
3	+500*XB		Z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Acreage		
4	ХВ		Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres
5	840		Z	=	\$420,000		X <sub>B</sub> =	600	acres
6	810								
7	780		Constrain	ts					
8	750		Constrain	t 1					
9	720		XA	≤	400				
10	690		=T4	≤					

Figure 4

	Q10	•	(= × 🗸 )	f <sub>*</sub> 400					
	М	N	0	Р	Q	R	S	Т	U
1			Objective	Function					
2	ctive		Maximize	Z			Variables		
3	⊦500*XB		Z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Acreage		
4	XB		Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres
5	840		Z	=	\$420,000		X <sub>B</sub> =	600	acres
6	810								
7	780		Constraint	ts					
8	750		Constraint	1					
9	720		XA	≤	400				
10	690		400	≤	400				



Constraint 2 specifies that the acreage of crop A (XA) must be equal to or greater than 0 acres. Link cell O13 with the value of XA (cell T4) by typing in cell O13 "=T4"; see Figure 6. Type in cell Q13 the constraint value of 0 (Figure 7).

TREND $\checkmark ( \land \checkmark \checkmark f_x = T4$												
	М	N	0	Р	Q	R	S	Т	U			
1			Objective	Function								
2	ctive		Maximize	Z			Variables					
3	+500*XB		z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Acreage					
4	ХВ		Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres			
5	840		Z	=	\$420,000		X <sub>B</sub> =	600	acres			
6	810											
7	780		Constraint	s								
8	750		Constraint	1								
9	720		X <sub>A</sub>	≤	400							
10	690		400	5	400							
11	660		Constraint	2								
12	630		X <sub>A</sub>	≥	0							
13	600		=T4	2								



Q13 $\checkmark (\bigcirc X \checkmark f_x \mid 0$											
_	М	N	0	Р	Q	R	S	Т	U		
1			Objective	Function							
2	ctive		Maximize	Z			Variables				
3	+500*XB		Z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Acreage				
4	XB		Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres		
5	840		z	=	\$420,000		X <sub>B</sub> =	600	acres		
6	810										
7	780		Constrain	ts							
8	750		Constrain	t 1							
9	720		XA	≤	400						
10	690		400	≤	400						
11	660		Constrain	t 2							
12	630		X <sub>A</sub>	≥	0						
13	600		400	2	0						

Figure 7

Constraint 3 specifies that the acreage of crop B ( $X_B$ ) must be equal to or less than 600 acres. Link cell O16 with the value of  $X_B$  (cell T5) by typing in cell O16 "=T5"; see Figure 8. Type in cell Q16 the constraint value of 600 (Figure 9).

	TREND $\checkmark ( X \checkmark f_x = T5$											
	М	N	0	Р	Q	R	S	Т	U			
1			Objective	Function								
2	ctive		Maximize	Z			Variables					
3	⊦500*XB		Z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Acreage					
4	XB		Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres			
5	840		Z	=	\$420,000		X <sub>B</sub> =	600	acres			
6	810											
7	780		Constrain	ts								
8	750		Constrain	t 1								
9	720		X <sub>A</sub>	≤	400							
10	690		400	≤	400							
11	660		Constrain	t 2								
12	630		X <sub>A</sub>	2	0							
13	600		400	2	0							
14	570		Constrain	t 3								
15	540		X <sub>B</sub>	≤	600							
16	510		=T5	≤								



	Q16	- (	🖲 🗙 🗸 J	£ 600								
	М	N	0	Р	Q	R	S	Т	U			
1			Objective	Function								
2	ctive		Maximize	Z			Variables					
3	+500*XB		Z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Acreage					
4	XB		Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres			
5	840		Z	=	\$420,000		X <sub>B</sub> =	600	acres			
6	810											
7	780		Constraint	ts								
8	750		Constraint	1								
9	720		X <sub>A</sub>	≤	400							
10	690		400	≤	400							
11	660		Constraint	2								
12	630		X <sub>A</sub>	2	0							
13	600		400	2	0							
14	570		Constraint	3								
15	540		X <sub>B</sub>	≤	600							
16	510		600	≤	600							
	Figure 9											

Constraint 4 specifies that the acreage of crop B ( $X_B$ ) must be equal or greater than 0 acres. Link cell O19 with the value of  $X_B$  (cell T5) by typing in cell O19 "=T5"; see Figure 10. Type in cell Q19 the constraint value of 0 (Figure 11).

	TREND $\checkmark$ ( $\checkmark$ $\checkmark$ $f_x$ =T5											
	М	N	0	Р	Q	R	S	Т	U			
1			Objective	Function								
2	ctive		Maximize	Z			Variables					
3	⊦500*XB		Z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Acreage					
4	XB		Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres			
5	840		z	=	\$420,000		X <sub>B</sub> =	600	acres			
6	810											
7	780		Constraint	ts								
8	750		Constraint	1								
9	720		X <sub>A</sub>	5	400							
10	690		400	≤	400							
11	660		Constraint	t <b>2</b>								
12	630		X <sub>A</sub>	2	0							
13	600		400	2	0							
14	570		Constraint	t 3								
15	540		X <sub>B</sub>	≤	600							
16	510		600	≤	600							
17	480		Constraint	t <b>4</b>								
18	450		X <sub>B</sub>	≥	0							
19	420		=T5	2								
				-	. 10							



	Q19 $\checkmark (f_x) = 0$												
	М	N	0	Р	Q	R	S	Т	U				
1			Objective	Function									
2	ctive		Maximize	Z			Variables						
3	+500*XB		Z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Acreage						
4	ХВ		Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres				
5	840		Z	=	\$420,000		X <sub>B</sub> =	600	acres				
6	810												
7	780		Constraint	s									
8	750		Constraint	:1									
9	720		X <sub>A</sub>	≤	400								
10	690		400	≤	400								
11	660		Constraint	2									
12	630		XA	2	0								
13	600		400	2	0								
14	570		Constraint	3									
15	540		X <sub>B</sub>	≤	600								
16	510		600	≤	600								
17	480		Constraint	4									
18	450		X <sub>B</sub>	2	0								
19	420		600	≥	0								
	Figure 11												

Constraint 5 specifies that the sum of the water use in crop A  $(3^*X_A)$  and crop B  $(2^*X_B)$  must be equal or less than 1800 acre-feet. In cell O22 we have to write this equation as follow: "=Z4\*T4+Z5\*T5"; see Figure 12. Type in cell Q19 the constraint value of 1800 (Figure 13).

SUM	Ŧ	: × 🗸	<i>f<sub>x</sub></i> =Z4*T4	1+Z5*T5											
	м	Ν	Ο	Р	Q	R	S	т	U	V	w	x	γ	Z	AA
1			Objective	Function											
2 0	tive		Maximize	Z											
3 ⊦	500*XB		z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Variables			Profit			Crop duty		
4	ХВ		Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres	C <sub>A</sub> =	\$300	/acre	Crop A=	3.0	AF/acre
5	840		z	=	\$420,000		X <sub>B</sub> =	600	acres	C <sub>B</sub> =	\$500	/acre	Crop B=	2.0	AF/acre
6	810														
7	780		Constrain	ts			1200 -						handa 4.2		
8	750		Constraint	1							nstraint 1	Cons	traint 2		
9	720		X <sub>A</sub>	≤	400		х	<u>_≥0</u>			nstraint 5	Ohio	ctivo		
10	690		400	≤	400		1000	<b>→</b>	X₄≤400		lution	Obje	cuve		
11	660		Constraint	2				ړدې	<b>~</b> ←	- 50	lation				
12	630		X <sub>A</sub>	≥	0		800	₹×5							
13	600		400	≥	0		s)								
14	570		Constraint	3			e l	63							
15	540		X <sub>B</sub>	≤	600		J <sup>600</sup>	X <sub>в</sub> ≤600			X <sub>в</sub> ≤600				
16	510		600	≤	600		_ V				1	1			
17	480		Constraint	4			400 -								
18	450		X <sub>B</sub>	2	0			Feasible							
19	420		600	≥	0			Region		wy tu					
20	390		Constraint	5			200			₹¥ ₹					
21	360		3X <sub>A</sub> +2X <sub>B</sub>	≤	1800				X <100	198					
22	330	=	Z4*T4+Z5*	Г5			0	$\xrightarrow{A^{\geq 0}}$	A_400	R 100	X <sub>в</sub> ≥0				
23	300						0	200	) 400	600	800	1000	1200		
24	270									X <sub>A</sub> (acre	es)				
25	240														
26															

Figure 12

	Q22 $\checkmark ( \checkmark \checkmark f_x )$ 1800									
	М	N	0	Р	Q	R	S	Т	U	
1			Objective	Function						
2	ctive		Maximize	Z			Variables			
3	+500*XB		Z	=	C <sub>A</sub> *X <sub>A</sub> +C <sub>B</sub> *X <sub>B</sub>		Acreage			
4	ХВ		Z	=	300*XA+500*XB		X <sub>A</sub> =	400	acres	
5	840		Z	=	\$420,000		X <sub>B</sub> =	600	acres	
6	810									
7	780		Constraint	s						
8	750		Constraint	1						
9	720		X <sub>A</sub>	≤	400					
10	690		400	≤	400					
11	660		Constraint	2						
12	630		X <sub>A</sub>	≥	0					
13	600		400	≥	0					
14	570		Constraint	3						
15	540		Х <sub>в</sub>	≤	600					
16	510		600	5	600					
17	480		Constraint	4						
18	450		X <sub>B</sub>	2	0					
19	420		600	≥	0					
20	390		Constraint	5						
21	360		3X <sub>A</sub> +2X <sub>B</sub>	5	1800					
22	330		2400	≤	1800					

Figure 13

Save your spreadsheet (Ctrl+S).

# **Defining the Optimization Linear Program**

Now, let's define the linear program model. Go to the Data menu and click on "Min Z  $x \le y x=2$  Model" icon (Figure 14)

Formulas Data Review View Developer Add-Ins DataUp Acrobat	
2↓       2↓ <t< td=""><td>je Model Ive ver ≁</td></t<>	je Model Ive ver ≁
Connections         Sort & Filter         Data Tools         Outline         Analysis         OpenSolve	

Figure 14

A dialogue window will appear, displaying the "Open Solver - Model" (Figure 15).

What is AutoModel?		AutoModel	
utoModel is a feature of OpenSolver he structure of the spreadsheet. It v penSolver or Solver. Note that you lease note that AutoModel will replace	that tries to automatically determin vill turn its best guess into a Solver don't have to use this feature: the te the model in this window, but wo	he the problem you are trying to optimise by the obser model, which you can then edit in this window and solv model can still be built manually. n't save it to the sheet until you click Save Model.	ring e wi
)bjective cell:	( maximise	C minimise C target value:	_
'ariable cells:			
onstraints:			_
<add constraint="" new=""></add>			•
		_	
		Add constraint Cancel	
		Delete selected constraint	
		✓ Make unconstrained variable cells non-nega	tive
1odify a constraint: select, make cha Idd a constraint: select "Add new cor	nges, then click "Update constraint" nstraint", enter the new constraint'	". s details, then click "Add Constraint"	

Figure 15

Go to the Objective cell section, and click on the icon to browse the objective cell (Figure 15). Select the cell Q5, which is the cell with the equation of the objective function (Figure 16).

H		J	К	L	M	N	0	Р	Q		
		m=	-1.5				Objective Function				
9			-	-	2	× D	Maximiz	Z			
Op	enSolv	/er - Mo	del		(B)		z	=	CA"XA+CB"X		
11	Maximiz	ze'!\$O\$5					Z	=	300"XA+500"XE		
			_		_		Z	=	\$360,000		
50	0	50	825	50	690	_					
100	0	100	750	100	660		Constrai	nts			
150	0	150	675	150	630		Constrain	:1			
	-					1.0	· · ·				

Figure 16

When you come back to the dialogue window, make sure to select "maximize" (Figure 17).

OpenSolver - Mod	el	-	and in such that	-	and the second s	<b>— X</b>
What is AutoMo	del?				AutoModel	
AutoModel is a feat the structure of the OpenSolver or Solv Please note that Au	ure of OpenSolver that tri e spreadsheet. It will turn er. Note that you don't ha utoModel will replace the m	es to automatic its best guess ir ive to use this f iodel in this wind	ally determine nto a Solver m feature: the m dow, but won'	the problem yo odel, which you odel can still be t save it to the s	ou are trying to optimi can then edit in this v built manually. sheet until you click Si	se by the observing window and solve with ave Model.
Objective cell:	'1 Maximize'!\$Q\$5	_ 6	maximise	$\bigcirc$ minimise	⊖ target value:	0
Variable cells:						_

Figure 17

Now click on the icon to choose the cells that will be the variables (Figure 18). Select cells T4 and T5 (Figure 19).

OpenSolver - Mo	del	3 march	Column Do	and the state	-	1		X		
What is AutoModel? AutoModel										
AutoModel is a feature of OpenSolver that tries to automatically determine the problem you are trying to optimise by the observing the structure of the spreadsheet. It will turn its best guess into a Solver model, which you can then edit in this window and solve with OpenSolver or Solver. Note that you don't have to use this feature: the model can still be built manually. Please note that AutoModel will replace the model in this window, but won't save it to the sheet until you dick Save Model.										
Objective cell: 11Maximize'!\$Q\$5										
Variable cells:	Variable cells:									
			Figure	e 18						
К	L M	N	O P	Q	R	S	Т	U		
m= -1.5		OŁ	ojective Fur	iction						
onstraint 5	Objective	Ma	aximiz Z			Variables				
OnenSalver	Madal	2	X	CA"XA+CB	Хв	Acreage				
Opensolver -	wodel	<u> </u>	-	300"XA+500"	ХB	× <sub>A</sub> =	200	acres		
'1 Maximize'!\$	T\$4:\$T\$5			\$360,000		X <sub>B</sub> =	600	acres		
0 750 -	100 000	6		10						



Now, we will add the constraints to the model. Let's start with the first constraint ( $X_A \leq 400$ ), go to the constraints section and click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O10 (Figure 21), then go back to the dialogue menu. Select the symbol of equal or less than " $\leq$ " (Figure 22). Now click on the icon to select the right side of the inequality (Figure 23), cell Q10 (Figure 24) and go back the dialogue window. Click on "Add Constraint" (Figure 25). The inequality Cell O10  $\leq$  Cell Q10 should have appeared in the left side of the Constraints section of the dialogue window (Figure 26). We just have finished declaring the first constraint.

OpenSolver - Model										
What is AutoModel? AutoModel AutoModel AutoModel the problem you are trying to optimise by the observing										
Automodel is a relate of operation of the standard relation of the structure of the spreadsheet. It will turn its best guess into a Solver mo OpenSolver or Solver. Note that you don't have to use this feature: the mo Please note that AutoModel will replace the model in this window, but won't	del, which you can then edit in this window and solve with del can still be built manually. save it to the sheet until you click Save Model.									
Objective cell: 1 Maximize'!\$Q\$5	C minimise C target value:									
Variable cells: 1 Maximize'!\$T\$4:\$T\$5	_									
Constraints:										
<add constraint="" new=""></add>										
	Add constraint Cancel									
	Delete selected constraint									
	Make unconstrained variable cells non-negative									

Figure 20

Н		J	К	L	М	N	0	Р	Q	B
		m=	-1.5		_		Objectiv	e Func	tion	
On	enSol	ver - Mo	del		2	X	Maximiz	Z		
-	choon		a.c.i	-		z	=	Ca"Xa+Ca"	Хв	
11	Maximi	ze'!\$0\$1	0				Z	-	300"XA+500">	КB
5	U	U	900	U	720		z	=	\$360,000	
50	0	50	825	50	690					
100	0	100	750	100	660		Constrai	ints		
150	0	150	675	150	630		Constrain	t1		
200	0	200	600	200	600		Xe	≤	400	
250	0	250	525	250	570		200	≤	400	
200	~	000	450	000	5.40			~		

Figure 21

[	'1 Maximize'!\$O\$10	_	=	•	
		_	= <=		
	Add constraint	Can	>= int bin alldif	Ŧ	
	Delete selected	constraint	Childhi		

Figure 22

OpenSolver - Model	
What is AutoModel?	AutoModel
AutoModel is a feature of OpenSolver that tries to automatically determin the structure of the spreadsheet. It will turn its best guess into a Solver of OpenSolver or Solver. Note that you don't have to use this feature: the r Please note that AutoModel will replace the model in this window, but wor	e the problem you are trying to optimise by the observing model, which you can then edit in this window and solve with model can still be built manually. n't save it to the sheet until you click Save Model,
Objective cell: 11 Maximize'!\$Q\$5	C minimise C target value:
Variable cells: '1 Maximize'!\$T\$4:\$T\$5	
Constraints:	
<add constraint="" new=""></add>	'1 Maximize'!\$O\$10 _ = = •
	Add constraint Cancel
	Delete selected constraint



Н		J	К	L	M	N	0	Р	Q	
		m=	-1.5				<b>Objective Fun</b>		tion	
Const	traint 4	Const	raint 5	Obje	Objective		Maximiz	Z		
XE	≥0	3XA+2>	(B≤1800	300°XA	300"XA+500"XB		Z =		Ca"Xa+CB"X	
XA	XB	XA	ХB	XA	XB		Z	=	300"XA+500"XI	
0	0	0	900	0	720		z	=	\$360,000	
50	0	50	825	50	690					
10-				1000	9	× D	Constrai	ints		
1 0	penSol	ver - Mo	odel		L I		Constrain	t1		
2 🗔	Maxim		ol		X <sub>A</sub>	≤	400			
킨나	Maxim	26 (ခုပ္ပခ္)	.ul			200	≤	400		
300	0	300	450	300	540		Constrain	+2		

Figure 24

OpenSolver - Mo	del	7		1.25	AutoModel	
AutoModel is a fe the structure of t OpenSolver or So Please note that	ature of OpenSolver that t he spreadsheet. It will turn lver. Note that you don't h AutoModel will replace the i	ries to automa its best guess ave to use this model in this w	tically determin i into a Solver r i feature: the r indow, but wor	e the problem you nodel, which you nodel can still be I't save it to the	ou are trying to optimi can then edit in this built manually, sheet until you click S	ise by the observing window and solve wit ave Model,
Objective cell:	'1 Maximize'!\$Q\$5	_	( maximise	() minimise	○ target value:	0
Variable cells:	'1 Maximize'!\$T\$4:\$T\$5					_
Constraints:						
<add cons<="" new="" td=""><td>straint&gt;</td><td></td><td></td><td>1 Maxi</td><td>mize'!\$Q\$10 mize'!\$Q\$10 d constraint</td><td>_ = V</td></add>	straint>			1 Maxi	mize'!\$Q\$10 mize'!\$Q\$10 d constraint	_ = V

Figure 25

OpenSolver - Model	×
What is AutoModel? AutoModel is a feature of OpenSolver that tries to automatically determine i the structure of the spreadsheet. It will turn its best guess into a Solver mo OpenSolver or Solver. Note that you don't have to use this feature: the mo Please note that AutoModel will replace the model in this window, but won't	AutoModel the problem you are trying to optimise by the observing del, which you can then edit in this window and solve with del can still be built manually. save it to the sheet until you dick Save Model.
Objective cell: ['1]Maximize'!\$Q\$5 _ ( maximise	C minimise C target value:
Variable cells: 1)Maximize'!\$T\$4:\$T\$5	_
Constraints:	
<add constraint="" new=""> \$O\$10 &lt;= \$Q\$10</add>	_     <=     ▼



To declare the second constraint ( $X_A \ge 0$ ), click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O13 (Figure 27), then go back to the dialogue menu. Select the symbol of equal or greater than " $\ge$ " (Figure 22). Now click on the icon to select right-side part of the inequality (Figure 23), select cell Q13 (Figure 29) and go back the dialogue window. Click on "Add Constraint". The inequality Cell O13  $\ge$  Cell Q13 should have appeared in the left side of the Constraints section of the dialogue window (Figure 30).

-1		J	К		M	N	Ο	Р	D I
		m=	-1.5	_			Objecti	e Func	tion
onst	raint 4	Const	raint 5	Obje	Objective		Maximiz	Z	
XE	≥0	3XA+2XB≤1800		300"XA+500"XB			z	=	CA'XA+CB'XB
:A	XB	XA	XВ	XA	XВ		Z	=	300"XA+500"XB
5	0	0	900	0	720		z	=	\$360,000
ю	0	50	825	50	690				
00	0	100	750	100	660		Constraints		
50	0	150	675	150	630		Constrain	it 1	
00	0	200	600	200	600		X <sub>A</sub>	≤	400
50	0	250	525	250	570		200	≤	400
20-		- 200	450	- 200			Constrain	it 2	
0	penSo	lver - M	odel		8	<u> </u>	×a	≥	0
16			4 al				200	≥	0
Constrain								it 3	
<u> </u>	-	-	450	FAA				•	000
					Figu	re 27			



-		J	К	L	M	N	0	Р	Q
		m=	-1.5				Objecti	ve Func	tion
onst	raint 4	4 Constraint 5		Obje	Objective		Maximiz	Z	
XE	≥0	3XA+2>	(B≤1800	300°XA	+500°XB		Z	=	CA"XA+CB"XB
(A	XВ	XA	XB	XA	XB		Z	=	300"XA+500"XB
0	0	0	900	0	720		z	=	\$360,000
50	0	50	825	50	690				
00	0	100	750	100	660		Constra	ints	
50	0	150	675	150	630		Constrain	nt 1	
00	0	200	600	200	600		Xe	≤	400
50	0	250	525	250	570		200	≤	400
ſo	penSc	lver - M	odel		2	x	Constrain	nt 2	
1				1000		X <sub>A</sub>	≥	0	
d   ':	1 Maxin	nize'!\$Q\$	13	200	≥	0			
<u>ل</u>	-		_				Constrain	nt 3	



penSolver - Mo	del	P 2	-	1 2	a ground	
What is AutoMo	odel?				AutoModel	
AutoModel is a fea the structure of th OpenSolver or Sol Please note that A	ature of OpenSolver that to ne spreadsheet. It will turn ver. Note that you don't h AutoModel will replace the r	ies to automa its best guess ave to use this nodel in this w	tically determin i into a Solver r into re: the r indow, but wor	e the proble nodel, which nodel can st n't save it to	m you are trying to optin you can then edit in this ill be built manually. the sheet until you click	nise by the observing s window and solve wi Save Model.
Objective cell:	'1 Maximize'!\$Q\$5	_	( maximise	() minimi	se 🔿 target value:	0
/ariable cells:	'1 Maximize'!\$T\$4:\$T\$5					
Constraints:						
<add cons<br="" new="">\$0\$10 = \$Q\$1</add>	:traint> D					_ >= •
\$0\$13 >= \$Q\$	13					-
					Add constraint	Cancel
					Delete selected	constraint

Figure 30

To declare the third constraint ( $X_B \le 600$ ), click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O16 (Figure 31), then go back to the dialogue menu. Select the symbol of equal or greater than " $\le$ " (Figure 32). Now click on the icon to select right-side part of the inequality (Figure 23), select cell Q16 (Figure 33) and go back the dialogue window. Click on "Add Constraint". The inequality of cell O16  $\le$  Cell Q16 should have appeared in the left side of the Constraints section of the dialogue window (Figure 34).

Н	1	J	K	L	M	N	0	Р	Q	
		m=	-1.5				Objectiv	ve Func	tion	
Cons	Constraint 4 Constraint 5		Obje	Objective		Maximiz	Z			
XE	3≥0	3XA+2>	<b≤1800< td=""><td>300°XA</td><td colspan="2">300"XA+500"XB</td><td>Z</td><td>=</td><td>CA"XA+CB"XB</td></b≤1800<>	300°XA	300"XA+500"XB		Z	=	CA"XA+CB"XB	
XA	XB	XA	XB	XA	XB		Z	=	300"XA+500"XB	
0	0	0	900	0	720		Z	=	\$360,000	
50	0	50	825	50	690				a	
100	0	100	750	100	660		Constraints			
150	0	150	675	150	630		Constrain	it 1		
200	0	200	600	200	600		X <sub>A</sub>	≤	400	
250	0	250	525	250	570		200	≤	400	
300	0	300	450	300	540		Constrain	it 2		
350	0	350	375	350	510		X <sub>A</sub>	≥	0	
100	0	400	300	400	480		200	2	0	
0	nenSol	ver - Mo	del		2	X	Constrain	it 3		
							X <sub>B</sub>	≤	600	
1	'1 Maximize'!\$O\$16 500 ≤									
	Constraint 4									

Figure 31





-		J	K	L	M	N	0	P	Q			
		m=	-1.5				Objectiv	e Fund	stion			
onst	onstraint 4 Constraint 5		raint 5	Objective			Maximiz	Z				
XE	≥0	3XA+2>	K <b>B≤1</b> 800	300°XA	+500°XB		z	=	CA"XA+CB"X			
:A	XB	XA	XВ	XA	XB		Z	=	300"XA+500"X			
)	0	0	900	0	720		z	=	\$360,000			
i0	0	50	825	50	690							
)0	0	100	750	100	660		Constrai	ints				
50	0	150	675	150	630		Constrain	t1				
D0	0	200	600	200	600		X <sub>A</sub>	≤	400			
50	0	250	525	250	570		200	≤	400			
D0	0	300	450	300	540		Constrain	t2				
50	0	350	375	350	510		X <sub>A</sub>	≥	0			
00	0	400	300	400	480		200	2	0			
Ío	penSc	olver - M	odel		2	X	Constrain	t3				
1.2		-		1000	-		X <sub>B</sub>	≤	600			
	1 Maxin	nize'!\$Q\$	16				600	600				
1	_	_	-	Constraint 4								



OpenSolver - Model										
What is AutoModel?	AutoModel									
AutoModel is a feature of OpenSolver that tries to automatically determine the problem you are trying to optimise by the observing the structure of the spreadsheet. It will turn its best guess into a Solver model, which you can then edit in this window and solve with OpenSolver or Solver. Note that you don't have to use this feature: the model can still be built manually. Please note that AutoModel will replace the model in this window, but won't save it to the sheet until you click Save Model.										
Objective cell: 1 Maximize'!\$Q\$5	C minimise C target value:									
Variable cells: 1'1 Maximize'!\$T\$4:\$T\$5	_									
Constraints:										
<add constraint="" new=""> \$0\$10 = \$Q\$10 \$0\$13 = \$Q\$13 \$<mark>\$0\$16 &lt;= \$Q\$16</mark></add>	Add constraint Cancel									

Figure 34

To declare the fourth constraint ( $X_B \ge 0$ ), click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O19 (Figure 35), then go back to the dialogue menu. Select the symbol of equal or greater than " $\ge$ " (Figure 36). Now click on the icon to select right-side part of the inequality (Figure 23), select cell Q19 (Figure 37) and go back the dialogue window. Click on "Add Constraint". The inequality Cell O19  $\ge$  Cell Q19 should have appeared in the left side of the Constraints section of the dialogue window (Figure 38).



Figure 35





Н	1	J	К	L	Μ	N	0	Р	Q	
		m=	-1.5				<b>Objective Fun</b>		ction	
onst	raint 4	Const	raint 5	Objective			Maximiz	Z		
XE	≥0	3XA+2XB≤1800		300"XA+500"XB			z	=	CA'XA+CB'X	
<a td=""  <=""><td>XB</td><td>XA</td><td>XВ</td><td>XA</td><td>XB</td><td></td><td>Z</td><td>=</td><td>300"XA+500"XE</td></a>	XB	XA	XВ	XA	XB		Z	=	300"XA+500"XE	
0	0	0	900	0	720		z	=	\$360,000	
50	0	50	825	50	690					
00	0	100	750	100	660		Constrai	nts		
50	0	150	675	150	630		Constraint 1			
00	0	200	600	200	600		Xe	≤	400	
50	0	250	525	250	570		200	≤	400	
00	0	300	450	300	540		Constraint	2		
50	0	350	375	350	510		X <sub>A</sub>	≥	0	
00	0	400	300	400	480		200	2	0	
50	0	450	225	450	450		Constraint	3		
00	0	500	150	500	420		Xe	≤	600	
7					9	× D	600	≤	600	
0	penSo	lver - M	odel		R		Constraint	4		
16	Maria						Xe	≥	0	
(Ľ	maxim	iize i şQş	tal				600	2	0	
SIL		750	-775	7511	270		Constraint	5		

Figure 37

OpenSolver - Mo	del	-	-	-	-	×		
What is AutoM	odel?		AutoModel					
AutoModel is a fea the structure of the OpenSolver or So Please note that a	ature of OpenSolver that trie he spreadsheet. It will turn it lver. Note that you don't hav AutoModel will replace the mo	es to autor is best gue ve to use t odel in this	natically determine ss into a Solver m his feature: the m window, but won	e the problem y nodel, which you nodel can still be 't save it to the	ou are trying to optim u can then edit in this e built manually. sheet until you click S	ise by the observing window and solve with Save Model.		
Objective cell:	'1 Maximize'!\$Q\$5	_	( maximise	() minimise	C target value:	0		
Variable cells:	'1 Maximize'!\$T\$4:\$T\$5					_		
Constraints:								
<pre><add cons<br="" new="">\$0\$10 = \$Q\$1 \$0\$13 &gt;= \$Q\$ \$0\$16 &lt;= \$Q\$ \$0\$19 &gt;= \$Q\$</add></pre>	straint> 0 \$13 \$16 \$19					_ >= ▼		
				Ad	d constraint	Cancel		
					Delete selected o	onstraint		
				🔽 Make	unconstrained variab	le cells non-negative		

Figure 38

To declare the fifth constraint  $(3X_A+2X_B \le 1800)$ , click on the icon to browse for the left hand side of the constraint (Figure 20). Select the cell O22 (Figure 39), then go back to the dialogue menu. Select the symbol of equal or greater than " $\le$ " (Figure 40). Now click on the icon to select rightside part of the inequality (Figure 23), select cell Q22 (Figure 41) and go back the dialogue window. Click on "Add Constraint". The inequality of cell O22  $\ge$  Cell Q22 should have appeared in the left side of the Constraints section of the dialogue window (Figure 42).

Н	- 1	J	К	L	Μ	N	0	Р	Q
		m=	-1.5				Objectiv	e Func	tion
Const	raint 4	Const	raint 5	Objective			Maximiz	Z	
XE	≥0	3XA+2>	<b≤1800< td=""><td>300°XA</td><td>+500°XB</td><td></td><td>z</td><td>=</td><td>CA"XA+CB"X</td></b≤1800<>	300°XA	+500°XB		z	=	CA"XA+CB"X
XA	XB	XA	XB	XA XB			Z	=	300"XA+500"XI
0	0	0	900	0	720		Z	=	\$360,000
50	0	50	825	50	690				
100	0	100	750	100	660		Constrai	nts	
150	0	150	675	150	630		Constraint	:1	
200	0	200	600	200	600		X <sub>A</sub>	≤	400
250	0	250	525	250	570		200	≤	400
300	0	300	450	300	540		Constraint	2	
350	0	350	375	350	510		X <sub>A</sub>	Σ	0
400	0	400	300	400	480		200	2	0
450	0	450	225	450	450		Constraint	3	
500	0	500	150	500	420		×e	≤	600
550	0	550	75	550	390		600	≤	600
600	0	600	0	600	360		Constraint	4	
650	0	650	-75	650	330		XB	2	0
700_		700	-150	700	300		600	≥	0
( Op	enSol	ver - Mo	del		2	x	Constraint	5	
_		-		-	-		3X <sub>A</sub> +2X <sub>B</sub>	≤	1800
1	Maximiz	ze'!\$0\$2	2				1800	< s	1800
A									





Figure 40

+	1	J	K	L	Μ	N	0	P	Q
		m=	-1.5				Objectiv	e Func	tion
onst	raint 4	Const	raint 5	Obje	ctive		Maximiz	Z	
XB	≥0	3XA+2XB≤1800		300"XA+500"XB			z	=	CA'XA+CB'
:A	XВ	XA	XB	XA	XB		Z	=	300"XA+500"X
)	0	0	900	0	720		Z	=	\$360,000
i0	0	50	825	50	690				
00	0	100	750	100	660		Constra	ints	
50	0	150	675	150	630		Constrain	e 1	
D0	0	200	600	200	600		X <sub>A</sub>	≤	400
50	0	250	525	250	570		200	≤	400
D0	0	300	450	300	540		Constrain	t 2	
50	0	350	375	350	510		X <sub>A</sub>	≥	0
00	0	400	300	400	480		200	2	0
50	0	450	225	450	450		Constrain	t3	
00	0	500	150	500	420		X <sub>B</sub>	≤	600
50	0	550	75	550	390		600	≤	600
D0	0	600	0	600	360		Constrain	t 4	
50	0	650	-75	650	330		X <sub>B</sub>	2	0
	-			1000	2	Y	600	2	0
0	penSo	lver - M	odel		Constrain	t 5			
1	Maxim	ize'l¢∩¢	22		$3X_{A}+2X_{B}$	≤	1800		
	- Piu Alli			1800	≤	1800			

Figure 41

OpenSolver - Mo	del	a di sana i sana i	x				
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Figure 42

Check the box that says "List constraints and shadow prices in a table with top left cell:" and select the cell "AC8" (Figure 43)

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Figure 43

Click on "Save Model" (Figure 44). A Series of colorful rectangles should have appeared in your Spreadsheet (Figure 45). Notice that the Objective Function (cell Q5) has a "Max" label on top of it. Also, notice that the variables have a pink rectangle on top. In addition, for each constraint the right and left side of the equations/inequality are related with a line and a symbol (" $\geq$ ", " $\leq$ " or "="), Double check that these symbols are correct.

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Figure 44



# **Running the Linear Programming Model**

It is time to run the model!!! Click on the Solve icon (Figure 46) of the Open Solver. After the screen blinks you will notice that numbers have changed. The maximum profit that can be obtained are \$360,000 (Cell Q5)!!! The values for the variable have changed,  $X_A=200$  and  $X_B=600$ , as we calculated in the presentation at class!!!



### To be turned in:

- 1) Take a look at the shadow values. For the constraint in cells O16 $\leq$ =Q16 (X<sub>B</sub> $\leq$ 600) the shadow value is equal to -300, what does it mean? Do the following to get a hint for the answer of this question: Change the value of cell Q16 to 601 and run the model. What is the new value of the objective function (cell Q5)? Now change it to 602, and take a look at the objective function value. How much the objective value changed for 601? And for 602? And for 603? Does the shadow price tell you how much the objective function will change by a unit increase in the constraint?
- 2) A screenshot of the model, like Figure 45.
- 3) Now let's invert the values so the Profits for  $C_A$  (cell W4) are equal to \$500/acre and for  $C_B$  (cell W5) are equal to \$300/acre. Click on Solve. What are the results for X<sub>A</sub> and X<sub>B</sub>?

Does the value of the profits change the results for the optimal solution? If so, do you think changing the market prices can change the optimal solution of linear systems?

4) A screenshot of the new model with the profit values changed.

# Exercise 2 (Adapted from Loucks and van Beek)

A city planner wants to know the minimum cost of waste removal (WR) at Sites 1 and 2 upstream of a recreational park (Figure 47). The cost of waste removal can be obtained by summing the cost of 100% ( $C_1$  and  $C_2$ ) times the fraction of removal for each site ( $X_1$  and  $X_2$ ). According to a predesign, the fraction of WR at site 1 ( $X_1$ ) must be equal to or greater than 0.8 but equal to or less than 1. The fraction of WR at site 2 ( $X_2$ ) must be equal to or less than 1. There is another constraint expressed as the equation:  $X_1 + 1.3 X_2 \ge 1.8$ .



Use the "Optimization" presentation in class to solve this problem. The optimization model is as follows:

Minimize Z  

$$Z = C_1(x_1) + C_2(x_2)$$
Where:  $C_1 = 200; C_2 = 100$   
Subject to  
 $x_1 \ge 0.8$   
 $x_1 \le 1$   
 $x_2 \ge 0$   
 $x_2 \le 1$   
 $x_1 + 1.3x_2 \ge 1.8$ 

Using the Excel spreadsheet Ex\_5.xlsx in Tab "2) Minimize", create a linear program using the Open Solver Model. Don't forget to select "<u>Minimize</u>" instead of "Maximize" (Figure 17) when creating the linear model.

#### To be turned in:

- 1) A screenshot of your linear model, such as Figure 44.
- 2) What is the minimum cost of removal (the optimum value of the objective function) obtained once the model has been run? How do your results compare with the ones in the presentation (Slide 20, minimum cost of removal equal to \$238K)

# Exercise 3 The perfect outfit! Extra credits (+20%)

Summer is coming and you are looking for the perfect outfit to wear, while, at the trying not to over spend. You have a total budget of \$130. This is a suggested number of minimum and maximum number of pieces that you want to buy for each article.

Article	Minimun # of pieces	Maximum # of Pieces
Jeans (C <sub>Jeans</sub> )	1	3
T-Shirts ( $C_{T-shirts}$ )	1	4

Other constraints are that you want to buy equal or more T-shirts than jeans ( $C_{T-shirts} \ge C_{Jeans}$ ), and that the total amount spent should not be more than your dedicated budget \$130.

### To be turned in:

Following the seven steps of linear programing (page 4 of this exercise)

- 1) Write down the *objective function* and the constraints
- 2) Draw the *feasible region*. You can use excel, or a simple piece of paper and scan it, or some engineering paper.
- 3) Obtain the vertices of the feasible region. Substitute the values of the vertices into the objective function. Submit a table showing the vertices of the feasible region as well as the value of the objective function for each pair of *C<sub>Jeans</sub>* and *C<sub>T-shirts</sub>* values (see slides 13 and 19 of the presentation of "Optimization").
- 4) Based on the previous analysis, what is the combination Jeans and T-shirts that you should buy?

Now, let's use the Open Solver model of excel to solve this problem. Use the file Ex\_5.xlsx - tab "3 Maximize" to create a linear optimization model. You can use as initial values for  $C_{Jeans}$  (cell G5) and  $C_{T-shirt}$  (cell G6) 1 and 1, respectively. Calculate amount spend in cell D6 (for the objective function) and B23 (for the constraint) by multiplying the cost of the jeans times the number of jeans (J5xG5) plus the cost of T=shirts times the number of T-shirts (J6xG6). When creating the linear optimization model in Open Solver, don't forget to select "Maximize" (Figure 17). In addition, you will have to declare in the optimization model that the number of jeans  $C_{Jeans}$  (cell G5) and T-shirts (cell G6) should be integers (see figure 48).

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Figure 48

- 5) Turn in a screenshot of your linear model, similar to Figure 44.
- 6) How much will you spend (the optimum value of the objective function) obtained once the model has been run (cell D5)?
- 7) How many Jeans and T-shirts you can buy *C<sub>Jeans</sub>* (cell G5) and *C<sub>T-shirts</sub>* (cell G6) for the maximum amount of money spent given the constraints?
- 8) How do these results compare with the same analysis done before in question 4 of this problem (see above)? Are these results similar?
- 9) What are the shadow prices for the different constraints? What is the maximum shadow price? If you change any constraint "1 unit", how much will the Objective function will change? Are there any biding constraints?

# Exercise 4 (Adapted from Loucks and van Beek) Extra credits (+20%)

Consider a water-using industry that plans to obtain water from a groundwater aquifer. Two wellfield sites have been identified, A and B. The objective of this industry is to minimize the cost of pumping from the wells at sites A and B. The cost of pumping can be obtained by summing the cost of pumping at site A plus site B (*Cost* A + Cost B); both of them depend on the water extracted  $Q_A$  and  $Q_B$  from sites A and B, respectively. Cost A is expressed by the equation:

$$Cost A = 8 + \frac{40 - 8}{17} * Q_A$$

Cost B is expressed by the equation:

$$Cost \ B = 15 + \frac{26 - 15}{13} * Q_B$$

For the well at site A, the water extracted  $(Q_A)$  must be equal to or greater than 0. The maximum sustainable groundwater extraction at site A is equal to or less than 17 acre-feet/year.

For the well at site *B*, the water extracted  $(Q_B)$  must be equal to or greater than 0. The maximum sustainable groundwater extraction at site *B* is equal to or less than 13 acre-feet /year. The water required to satisfy the industry is 22 acre-feet/year.



Figure 49

#### To be turned in:

Following the seven steps of linear programing (page 4 of this exercise)

- 1) Write down the *objective function* and the constraints
- 2) Draw the *feasible region*. You can use excel, or a simple piece of paper and scan it, or some engineering paper.
- 3) Obtain the vertices of the feasible region. Substitute the values of the vertices into the objective function. Submit a table showing the vertices of the feasible region as well as the

value of the objective function for each pair of  $Q_A$  and  $Q_B$  values (see slides 13 and 19 of the presentation of "Optimization").

- 4) Based on the previous analysis, what is the minimum cost of pumping for this industry?
- 5) What is the optimal water extraction  $Q_A$  and  $Q_B$  for the minimum cost of pumping?

Now, let's use the Open Solver model of excel to solve this problem. Use the file  $Ex_5.xlsx - tab$  "4 Minimize" to create a linear optimization model. You can use as initial values for QA (cell G5) and QB (cell G6) 10 and 10 AF/year, respectively. Calculate *Cost A* in cell D8 using the formula provided above. Similarly, calculate *Cost B* in cell D11 using the formula provided above. Don't forget to link the objective function in cell D5 by summing the *Cost A* (Cell D8) plus *Cost B* (Cell D11). When creating the linear optimization model in Open Solver, don't forget to select "Minimize" instead of "Maximize" (Figure 17).

- 6) Turn in a screenshot of your linear model, similar to Figure 44.
- 7) What is the minimum cost of pumping (the optimum value of the objective function) obtained once the model has been run (cell D5)?
- 8) What are the optimal water extractions  $Q_A$  (cell G5) and  $Q_B$  (cell G6) for the minimum cost of pumping?
- 9) How do these results compare with the same analysis done before in questions 4 and 5 of this problem (see above)? Are these results similar?