Uncertainty Propagation of Hydrologic Modeling in Water Supply System Performance: Application of Markov Chain Monte Carlo Method

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Abstract: It is imperative for cities to develop sustainable water management and planning strategies in order to best serve urban communities that are currently facing increasing population and water demand. Water resources managers are often chastened by experiencing failures attributed to natural extreme droughts and floods. However, recent changes in water management systems have been responding to these uncertain conditions. Water managers have become thoughtful about the adverse effects of uncertain extreme events on the performance of water supply systems. Natural hydrologic variability and inherent uncertainties associated with the future climate variations make the simulation and management of water supplies a greater challenge. The hydrologic simulation process is one of the main components in integrated water resources management. Hydrologic simulations incorporate uncertain input values, model parameters, and a model structure. Therefore, stochastic streamflow simulation and prediction, and consideration of uncertainty propagation on performance of water supply systems (WSSs) are essential phases for efficient management of these systems. The proposed integrated framework in this study models a WSS by taking into account the dynamic nature of the system and utilizing a Markov chain Monte Carlo (MCMC) algorithm to capture the uncertainties associated with hydrologic simulation. Hydrologic responses from the results of a rainfall-runoff model for three watersheds of Karaj, Latyan, and Lar in Tehran, Iran, as the case study are used as inputs to the reservoirs. Results confirm that uncertainties associated with the hydrologic model’s parameters propagate through the simulation and lead to a wide variation in reservoir storage and WSS performance metrics such as vulnerability and reliability. For example, water storage simulation in the Karaj reservoir can vary up to 70% compared with the observed values. This causes contradiction and conflict in the management of reservoirs and water systems and decision making. The results emphasize the importance of analyzing WSS performance under uncertain conditions to improve the simulation of natural processes and support water managers for a more efficient decision-making process. DOI: 10.1061/(ASCE)HE.1943-5584.0001646, © 2018 American Society of Civil Engineers.

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Introduction

The stationarity has hitherto been the main assumption for the operation and management of water systems (Milly et al. 2008). Water supply systems (WSSs) are designed to deliver water to communities over the length of the design period and are based on archaic rules. Changes in hydroclimatology and infrastructures affect WSSs, thus water managers should consider these changes for further risk assessment and planning. To capture the effect of hydroclimatic uncertainty propagation on water supply systems' performance, there is a need to develop integrated stochastic models. To support the decision-making process and capture uncertainties in integrated water resources management, integrated approaches have been developed to combine water system models (e.g., Goharian et al. 2016) along with the performance assessment concepts (e.g., Hashimoto et al. 1982) for water supply systems (e.g., Fowler et al. 2003; Ajami et al. 2008).

Simulating the physics of the formation and movement processes of water on Earth is a leading phase in the integrated planning and management of WSSs. The uncertainty in the amount and timing of inflow to the system can be presented by incorporating the full range of streamflow variation simulated by a hydrologic model (Georgakakos et al. 2004). However, study of the uncertainties associated with the hydrologic model parameters is not often taken into consideration (Vrugt et al. 2003). Different approaches are suggested for analyzing parameters' uncertainty in the hydrological modeling (Kitanidis and Bras 1980; Moradkhani et al. 2005; Ajami et al. 2007).

The Monte Carlo (MC) method has gained a great acceptance among researchers for uncertainty analysis because it has various benefits over conventional uncertainty analysis methods (Papadopoulos and Yeung 2001). This method is often used in the evaluation of complex environmental systems, such as models used in hydrology or ecology. Marton et al. (2011) used the Monte Carlo approach to estimate the uncertainty of average monthly streamflow discharges from the hydrologic model into the reservoir system. The Monte Carlo approach was later combined with Markov chain, called Markov chain Monte Carlo (MCMC), to
The Tehran, Iran, metropolitan region was selected as the study area to test the application of the proposed framework. Tehran is the capital and the largest city in Iran and has a population of more than 8 million people based on the 2010 census (Statistical Centre of Iran 2015). The city has an area of 730 km², located at the latitude of 35°31' to 35°57' N and longitude of 51°42' to 51°47' E. The average temperature is approximately 17°C. July is the hottest month with a mean minimum temperature of 26°C and mean maximum temperature of 36°C. January is the coldest month with a mean minimum temperature −1°C and mean maximum temperature 8°C. Most of the precipitation occurs from late autumn to mid-spring. The average annual precipitation is approximately 232 mm (Statistical Centre of Iran 2015).

Tehran’s WSS consists of three surface reservoirs including Karaj (in the west), Lar, and Latyan (in the east) (Fig. 1):

- Karaj Dam was constructed on the Karaj River in 1961. It supplies approximately 30% of the municipal water demand in Tehran and provides water for the irrigation area of approximately 500 km² near the city of Karaj. The annual average inflow to the reservoir is 472 million m³ (MCM) and the maximum capacity of the reservoir is 202 MCM.
- Latyan Dam was built on Jajrood River in 1967. This one of Tehran’s main sources of water supply. The Latyan Reservoir meets the irrigation water demand of approximately 300 km² of farms in this region. The annual average inflow to the Latyan reservoir is 245 MCM and the maximum capacity of the reservoir is 95 MCM.
- Lar Dam was constructed on the Lar River in 1982. The primary objective behind building this reservoir was to supply Tehran’s increasing municipal water demand during the 1970s and 1980s. Moreover, this reservoir helps fulfill the irrigation water demand in this region. The annual average inflow to the reservoir is 481 MCM and maximum capacity of the reservoir is 900 MCM.

The information regarding Tehran’s water supply system is collated from Tehran Province Water & Wastewater (ABFA) website (2017) and Karamouz et al. (1999).

In addition to the three main reservoirs, Taleghan Reservoir, with a capacity of approximately 420 MCM in the west part of Tehran, has been operating since 2006. This reservoir is deemed of supplying approximately 278 MCM water for irrigation demand, 150 MCM for domestic water demand, and 12 MCM for environmental and recreational purposes. Ideally, the surface water system provides up to 800 MCM of water, 350 MCM by the Lar and Latyan reservoirs, 300 MCM from the Karaj Reservoir, and 150 MCM by the Taleghan Reservoir. Because of a lack of information about the Taleghan Reservoir operation, such as streamflow data for calibration, validation, and uncertainty assessment, the time period for this study was selected to be before the operation of Taleghan Reservoir started (i.e., 2001–2006). In addition to surface water resources, water can be withdrawn from the groundwater via deep wells that are distributed throughout the system. The total maximum allowable discharge from these wells is limited to 250 MCM. However, based on recent records, groundwater overdraft in Tehran has been exceeded by up to 370 MCM during the dry periods (Iran Ministry of Energy 2009).

The Tehran WSS has five water treatment plants (WTPs) with a maximum capacity of 19 m³/s. Karaj Reservoir provides water for WTPs 1 and 2 with the total capacity of 11.5 m³/s. The water is transferred from Latyan Reservoir to WTPs 3 and 4 through the Teloo Tunnel with the maximum capacity of 10 m³/s. Water Treatment Plant 5 also receives water from the Lar Reservoir.
water through deep wells is added into the water distribution network after meeting the required standards for treatment. A schematic of Tehran’s WSS is shown in Fig. 1. The average water consumption of Tehran is approximately 300 L per capita per day (Iran Ministry of Energy 2013). The watersheds and location of the reservoirs are illustrated in Fig. 2. The Karaj Reservoir watershed has an area of 874 km². The watershed area for the Latyan, Lar, and Taleghan reservoirs are 435, 675, and 960 km², respectively.

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Methodology

An integrated framework is proposed to model the water supply system and evaluate its performance by considering the uncertainties in streamflow simulation. The proposed methodology includes four main steps shown in Fig. 3.

Data Collection

Historical data, from the Iran Water Resources Management Organization (2018) database, were collected and used to develop the rainfall-runoff simulation model. Daily precipitation, temperature, and streamflow during the period of 2001–2006 were gathered for the nearest synoptic stations to the centroid of the Karaj, Latyan, and Lar watersheds. This time period serves as a sample time period for conceptual purposes. Moreover, this time period has sufficient and reliable data for the application of the proposed framework, and subsequently further research can be straightforwardly performed for other periods and wet, dry, and normal years.

Streamflow Simulation

The HBV model is a conceptual numerical model of the watershed physics and hydrological process that simulates runoff mainly using temperature \( T \), potential evapotranspiration, and precipitation inputs (Bergström et al. 1992). Different routines in the model can represent the snowmelt and runoff by using the degree-day method, soil moisture, evaporation, and groundwater. The three main linear reservoir equations and channel routing by a triangular weighting function form the core of the model. The general water balance in HBV is as follows:

\[
P - E - Q = \frac{d(SP + SM + UZ + LZ + \text{lakes})}{dt}
\]

where \( P \) = precipitation; \( E \) = evapotranspiration; \( Q \) = runoff; \( SP \) = snow pack; \( SM \) = soil moisture; \( UZ \) = upper groundwater zone; \( LZ \) = lower groundwater zone; and \( \text{lakes} \) = reservoir volume. Groundwater recharge and actual evaporation are functions of actual water storage in a soil box and the runoff formation is represented by linear reservoir equations. Potential evaporation on day \( t \), \( E_{\text{pot}}(t) \), is calculated as

\[
E_{\text{pot}}(t) = E_{\text{pot,M}}(t) + C_{KT}(T(t) - T_{M})
\]

where \( T(t) \) = temperature in day \( t \); \( T_{M} \) = monthly mean temperature; \( E_{\text{pot,M}} \) = monthly mean potential evaporation; and \( C_{KT} \) = correction factor obtained through the model calibration (Bergström et al. 1992). Table 1 contains the list of the model parameters that are considered for the uncertainty analysis.

Uncertainty-Based Hydrologic Modeling

Uncertainty-based analysis of rainfall runoff can be used to determine the possible streamflow ranges and potential extreme events in the watershed. To estimate the values of the model parameters, calibration against historical observations is performed. A rainfall-runoff model \( (M) \) can be expressed as

\[
y = M(I, \theta)
\]

where \( y \) = watershed streamflow; \( \theta \) = set of the model parameters; and \( I \) = model input set.

Uncertainty Analysis Using MCMC Method

Monte Carlo simulation explains and transfers the propagation of uncertainties in the hydrologic model parameters into the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Initial range</th>
</tr>
</thead>
<tbody>
<tr>
<td>DD</td>
<td>Degree-day factor ( (\text{mm} \degree \text{C}^{-1} \text{day}^{-1}) )</td>
<td>3–7</td>
</tr>
<tr>
<td>K0</td>
<td>Near-surface-flow storage constant</td>
<td>0.05–0.2</td>
</tr>
<tr>
<td>K1</td>
<td>Interflow storage constant ( (\text{day}^{-1}) )</td>
<td>0.01–0.1</td>
</tr>
<tr>
<td>K2</td>
<td>Base flow storage constant ( (\text{day}^{-1}) )</td>
<td>0.01–0.05</td>
</tr>
<tr>
<td>Kp</td>
<td>Percolation storage constant ( (\text{day}^{-1}) )</td>
<td>0.01–0.05</td>
</tr>
<tr>
<td>FC</td>
<td>Maximum value of soil moisture</td>
<td>100–200</td>
</tr>
<tr>
<td>L</td>
<td>Storage (mm)</td>
<td>2–5</td>
</tr>
<tr>
<td>BETA</td>
<td>Threshold water level for near-surface flow (mm)</td>
<td>1–7</td>
</tr>
<tr>
<td>C</td>
<td>Shape coefficient</td>
<td>0.01–0.07</td>
</tr>
<tr>
<td>PWP</td>
<td>Soil permanent wilting point (mm)</td>
<td>90–180</td>
</tr>
</tbody>
</table>

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uncertainties in the model output (i.e., streamflow). Monte Carlo is a computational algorithm that generates samples of a parameter from a defined probability distribution. In the first step, a domain of all possible values for the parameter is defined. Then, a parameter is randomly generated from a prior probability distribution that is specified over the domain. Here, the prior distribution is deemed to be uniform considering the lack of available information for parameters.

After parameterizing the probability distributions for each parameter, MCMC sampler is utilized with the purpose of developing the Markov chain with predefined probability distributions for the parameters (Hastings 1970). Application of MCMC assists in estimating the posterior probability distribution function (PDF) for the parameters.

Supposedly, discrete parameter \( X \) (a parameter from parameters’ set \( \theta \)) has a probability density function of \( f_X(x) \) on set \( A \). The expected value of function \( g \) for \( X \) can be shown as

\[
E(g(x)) = \sum_{x \in A} g(x) f_X(x)
\]

(4)

For \( n \) samples of \( X \), \( (x_1, \ldots, x_n) \) with the mean of \( g(x) \), the MC estimate of \( E(g(x)) \) is expressed by

\[
\hat{g}_n(x) = \frac{1}{n} \sum_{i=1}^{n} g(x_i)
\]

(5)

If \( E(g(x)) \) exists for any arbitrarily small \( \varepsilon \), then

\[
\lim_{n \to \infty} P(|\hat{g}_n(x) - E(g(x))| \geq \varepsilon) = 0
\]

(6)

This means that as \( n \) gets larger, there is a small chance of \( g(x) \) deviating from \( E(g(x)) \). In the condition where \( n \) is large enough, \( \hat{g}_n(x) \) delivered from the Monte Carlo experiment would be closer to \( E(g(x)) \) (Anderson and Anderson 1999).

A sequence of random values of parameter \( X \) (i.e., \( X_1, X_2, \ldots, X_n \)) is a Markov chain if the conditional distribution of \( X_{n+1} \) given \( X_1, \ldots, X_n \) only depends on \( X_n \). If the state space of the Markov chain (which means the set where \( X_i \) occurs) is finite, then the initial distribution can be assigned with vector \( \lambda = (\lambda_1, \ldots, \lambda_n) \) so that

\[
Pr(X_1 = x_i) = \lambda_i i = 1, \ldots, n
\]

(7)

The transition probabilities can be stated with the use of matrix \( P \)

\[
Pr(X_{n+1} = x_j|X_n = x_i) = p_{ij} i = 1, \ldots, n \quad \text{and} \quad j = 1, \ldots, n
\]

(8)

where \( p_{ij} \) = element of matrix \( P \).

The DREAM algorithm (Vrugt et al. 2008) is employed to estimate the posterior PDFs of the parameters. The posterior PDFs are obtained by utilizing a likelihood function and the prior distribution based on the Bayesian approach. Likelihood function in MCMC sampling, as a formal Bayesian approach of DREAM, is used to correct the parameters’ prior distributions to converge to the posterior distribution (Vrugt et al. 2008). The model simulation error, \( e \), is obtained by

\[
e(\theta) = \tilde{y} - y(\theta)
\]

(9)

where \( y(\theta) \) and \( \tilde{y} \) = simulated and observed streamflow, respectively. Accounting for the uncertainty associated with the parameters, the model is calibrated by choosing a set of parameters that results in the minimum simulation error. This paper’s assumption (tested by using the skewness test of normality and autocorrelation and partial autocorrelation estimations) is that the errors are random and follow a Gaussian probability distribution. For the Gaussian likelihood, the posterior PDF can be estimated using the additive simple least-squares (SLS) objective function (\( F_{SLS} \)). When computing the PDFs of the parameters, the following density function is used:

\[
\pi(\theta|S) \propto c \times \pi(\theta) \times F_{SLS}(\theta|S)^{-\pi/2}
\]

(10)

where \( c = \) normalizing constant; \( \pi(\theta) = \) prior distribution of \( \theta \); and \( S = \) observed streamflow (composed of \( n \) events). The preceding distribution combines the parameter’s likelihood with its prior distribution according to the Bayesian theory.

**Performance Evaluation of Streamflow Simulation**

The following evaluation criteria are used to test the performance of the HBV model in streamflow simulation:

*Mean bias error:* \( MBE = (n)^{-1} \sum_{i=1}^{n} (S_i - O_i) \)

(11)

*Mean squared error:* \( MSE = (n)^{-1} \sum_{i=1}^{n} (S_i - O_i)^2 \)

(12)

*Common mean correlation:* \( CMC = \frac{\sum_{i=1}^{n} (S_i - \bar{S})(O_i - \bar{O})}{\sqrt{\sum_{i=1}^{n} (S_i - \bar{S})^2 \sum_{i=1}^{n} (O_i - \bar{O})^2}} \)

(13)

where \( O_i \) and \( S_i \) = observed and simulated streamflow in time step \( i \), respectively; \( \bar{O} \) and \( \bar{S} \) = mean values of observed and simulated streamflow; \( MBE = \) measurement of accuracy that depicts the difference between the expected value of runoff (simulation) and its true value (observation); \( MBE \) could be negative or positive, but the values closer to zero are preferred; \( MSE \), the second moment of the errors, measures the average of the squares of the errors or deviations, which is the difference between the observed and simulated runoff, and is nonnegative, and values closer to zero are also preferred; and CMC provides a tool for measuring the simulation performance and varies between 0 and 1 for weak and perfect simulation, respectively (Iliadis et al. 2007).

**Development of the SD Model**

The main objective of a WSS is to provide reliable water for various water demands, such as municipal, agricultural, and industrial. A simulation model provides an abstract representation of the WSS in order to understand the key factors that control the system, predict the future behavior of the WSS under changing conditions, and test alternative policies. To evaluate the overall system performance, interrelations of different components should be identified and dynamically modeled via the SD approach.

**Causal Loops and Relationships**

The first step in creating a SD model is to draw the causal loops and their relationships (Fig. 4). Casual loops can have either positive (reinforcing) or negative (balancing) relationships and effects. In Fig. 4, the key governing relationships for the Tehran WSS
are graphically shown. The plus signs above the tail of arrows show the positive effect, while the negative relationships are depicted by minus signs. The relationships and loops are drawn graphically using causal loop diagrams (CLDs), which helps provide a better understanding of the interrelationships and interactions between different system components graphically. As a general rule for a WSS, more precipitation in a system means an increase in the different system components graphically. As a general rule for a understanding of the interrelationships and interactions between minus signs. The relationships and loops are drawn graphically us-

Fig. 4. Casual loop diagram of Tehran water supply system corresponding to the long-term behavior of the reservoirs

are graphically shown. The plus signs above the tail of arrows show the positive effect, while the negative relationships are depicted by minus signs. The relationships and loops are drawn graphically using causal loop diagrams (CLDs), which helps provide a better understanding of the interrelationships and interactions between different system components graphically. As a general rule for a WSS, more precipitation in a system means an increase in the watersheds’ streamflow volume. This increase affects the down-
stream by raising the water level in the reservoir. Because the reser-
voir maximum storage restricts storing more water in the system, a larger capacity is needed within the system. The added capacity can be achieved by constructing a new infrastructure or expanding the existing reservoir’s capacity. New developments cause popula-
tion growth in the region, which leads to an increase in water de-
mand. The relationships among the components in Tehran’s water system CLD are conceptually presented in Fig. 4.

After creating the CLD, the diagram should be converted to a stock and flow diagram (SFD) in STELLA. A SFD is more a de-
tailed diagram that includes all the pieces and modules of the water system. The WSS of Tehran is modeled based on the following six components:

1. Surface water, which includes the following submodules:
   a. The Karaj Dam and WTPs 1 and 2;
   b. The Latyan Dam and WTPs 3 and 4; and
   c. The Lar Dam and WTP 5.
2. Groundwater;
3. Water demand;
4. Water resources allocation;
5. Development plan (incorporating Taleghan Dam in the WSS); and

Underlying Equations of SD Modeling
The surface water module of the water supply system includes the model of three individual reservoirs of Karaj, Latyan, and Lar and the linkages between them. While groundwater is assumed as an external resource, water supply from the groundwater is limited by a maximum allowable drawdown rate. For each of the surface reservoirs, the mass balance equation is employed

\[ \forall i: \frac{d\text{water}_i}{dt} = \text{inflow}_i - \text{outflow}_i \]  (14)

where \(d\text{water}_i\) = variation of storage in reservoir \(i\) in month \(t\); \(\text{inflow}_i\) = inflow to the reservoir \(i\) in month \(t\); and \(\text{outflow}_i\) = total release from reservoir \(i\) in month \(t\), which consists of three parts

\[ \text{outflow}_i = \text{FUi}_i + \text{FAi}_i + \text{FLi}_i \]  (15)

where \(\text{FUi}_i\) and \(\text{FAi}_i\) = water releases for domestic and agricultural demands, respectively; and \(\text{FLi}_i\) = total water loss from reservoir.

To represent and define water allocation policies among the demands, the higher priority is given to the municipal water need for Tehran. Whenever there is a water deficit in the surface water sys-
tem, withdrawal from groundwater resources can increase up to 50% more than the maximum capacity withdrawal rate (250 MCM per year). This prioritization is derived based on the current operation policies of the Tehran Province Water & Wastewater. The value of \(\text{FUi}_i\) is limited to the maximum structural capacity of the reservoir release and the transfer canal capacity from reservoirs to the WTPs. Variable \(\text{FUi}_i\) includes two parts: \(\text{FNI}_i\) and \(\text{FAi}_i\), which are the supply of water demand for domestic use under normal conditions and under water shortage, respectively.

In each month and reservoir, \(\text{FNI}_i\) is distributed proportionally between reservoirs based on the available stored water at the beginning of the month

\[ \text{FNI}_i = (\text{TUD}_i - \text{FGWw}) \times \frac{\text{V}_i}{\sum_{i=1}^{k} \text{V}_i} \]  (16)

where \(k\) = total number of reservoirs in the WSS; \(\text{V}_i\) = available volume of water in time \(t\) in reservoir \(i\); \(\text{TUD}_i\) = total domestic water demand in month \(t\); and \(\text{FGWw}\) = allowable discharge from the groundwater resources. The minimum reservoir water volume, \(V_{\text{min}}\), and reservoir maximum storage, \(V_{\text{max}}\), are subjected to the structural properties of the reservoir, and are included in a constraint

\[ \text{V}_{\text{min}} \leq \text{V}_i \leq \text{V}_{\text{max}} \]  (17)

Variable \(\text{FAi}_i\) has a specific value for each month (equally distributed throughout the month) and each reservoir, and is limited by the reservoir minimum storage

\[ \text{if} \quad \text{V}_i > \text{V}_{\text{min}} \quad \text{then} \quad \text{FAi}_i = \text{Pattern(month)}/i \quad \text{else} \quad \text{FAi}_i = 0 \]  (18)

where \(\text{Pattern(month)}\) = agricultural water demand in month \(t\) from reservoir \(i\).

Considering changes in groundwater storage, the area of the aquifer (496 km²), and storage coefficient 0.06, the groundwater level changes for each time step are calculated as

\[ \Delta \text{hg}_t = \Delta \text{VG}_t/(496 \times 0.06) \]  (19)

where \(\Delta \text{hg}_t\) = groundwater level change; and \(\Delta \text{VG}_t\) = groundwater volume change during month \(t\). More detail about the modeling of the system can be found in Goharian (2012). Considering the availability of historical data and model limitations, the daily time step is used for simulation. The model is calibrated for a 5-year period from September 23, 2001 to September 23, 2006.

Water Supply System Performance Evaluation
Different performance metrics including reliability, resiliency, and vulnerability (RRV) are selected to evaluate the WSS performance. Karamouz et al. (2012, 2013) evaluated the reliability of part of Tehran’s WSS under climate change conditions using reliability (\(\alpha\)) as the probability of nonoccurrence of failure during a certain time period. The reliability of a system in general is calculated as follows (Hashimoto et al. 1982):

\[ \text{Reliability} = \alpha \]
\[
\alpha = \text{Prob}[X_t \in S] \forall t
\]

where \( S \) = set of the system desired outputs. In this paper, to calculate the reliability \( (\text{Rel}) \), the following equation is employed:

\[
\text{Rel} = \frac{n_{\text{succ}}}{n_{\text{total}}}
\]

where \( n_{\text{succ}} \) = number of days where the system has success (satisfactory) status; and \( n_{\text{total}} \) = total number of the days in the simulation period. Success is defined as the ability of the system to supply water from surface and groundwater sources without exceeding the maximum allowable level of groundwater withdrawal. So, failure can be defined as any overdraft event from the groundwater to supply demands.

Resiliency index \( (\text{Res}) \) describes how quickly a system can recover from failure. If \( S \) denotes the set of satisfactory states and \( F \) the set of unsatisfactory states, then in the long run the number of transitions from satisfactory states in \( S \) to unsatisfactory states in \( F \) must be equal to the number of transitions in the reverse direction. Resilience \( (\beta) \) is defined as the conditional probability of a recovery from the failure set in a single time step (Hashimoto et al. 1982)

\[
\beta = \text{Prob}[X_{t+1} \in S | X_t \in F]
\]

where \( \beta \) is the fraction of days where the system has success (i.e., severity), which is the classical definition of vulnerability (Hashimoto et al. 1982)

\[
Vul = \frac{V_{\text{Fail}}}{TDD}
\]

where \( V_{\text{Fail}} = \) water shortage, which should be supplied with the excess groundwater withdrawal; and \( TDD = \) total water demand.

**Results**

**Uncertainty Analysis of the HBV Model Parameters**

Using DREAM, multiple chains were simultaneously run and the orientations of the parameters’ distributions were automatically tuned during the evolution to the posterior distributions. For each set of the parameters, the HBV model (linked with the MCMC algorithm using a MATLAB interface) was run to simulate streamflow. For each watershed, the model parameters’ PDFs were constructed. The upper and lower ranges of the parameters were then revised by investigating their domain of variations using the resultant PDFs. Hence, for some parameters the initial ranges are narrowed to set the proper lower or upper limits. Table 2 reports the model parameters’ means, standard deviations, and revised ranges. By comparing the parameter ranges in Table 2 and the initial values reported in Table 1, it can be concluded that the ranges of the model parameters need to be revised for each individual watershed.

Samples of generated parameters interpreted the behavior of the parameters’ distribution. For each watershed, 10,000 sets of parameters were generated within the revised ranges. The PDFs for parameters were built using parameters that met the MCMC algorithm convergence criteria. Fig. 5 is an example of the behavior of parameters’ distribution, in which PDFs of three parameters of HBV model including \( K_1, DD, \) and \( K_2 \) are illustrated for Karaj, Lar, and Latyan watersheds, respectively.

Based on the results, PDFs are well defined regarding the revised values of parameters. It reveals that the historical observed streamflow includes sufficient information to estimate the model parameters. The histogram of \( K_1 \) concentrates most of the probability mass at the lower bound of the parameter. This behavior of the histogram (highest probability mass either at the upper or lower bounds) could be attributed to the deficiencies in the model structure or errors in the input data. It even may raise the question of whether the parameter is actually representing the behavior of the watershed (Vrugt et al. 2008). The HBV model estimates runoff based on the reservoir concept: two conceptual reservoirs, one above the other. The upper reservoir is used to model the near-surface flow, while the lower one simulates the base flow. Recession coefficient \( K_1 \) represents the response function of the upper reservoir. Therefore, the observed streamflow, as the model input, directly affects the estimation of \( K_1 \) and its histogram.

In order to make a visual depiction of the distribution of the HBV model parameters and assess the location, dispersion, and symmetry or skewness of sets of parameters, box plots are presented in Fig. 6. Box plots compare the features in parameter sets for the study of watersheds.

Furthermore, box plots depict the range and distribution characteristics of the same parameter in different watersheds that could otherwise be quite different (e.g., parameter FC). Some of the parameters are well defined, while others have considerable uncertainty. Comparing the box plots in Fig. 6 demonstrates that although some of the parameters are equally distributed in their quartiles, parameters that are distributed with significant skewness tend to their lower or upper values. This considerable difference in

<table>
<thead>
<tr>
<th>Watershed</th>
<th>Parameter</th>
<th>( DD ) (mm °C(^{-1}) day(^{-1}))</th>
<th>FC (mm)</th>
<th>BETA (mm)</th>
<th>C</th>
<th>( K_0 ) (day(^{-1}))</th>
<th>L (mm)</th>
<th>( K_1 ) (day(^{-1}))</th>
<th>K2 (day(^{-1}))</th>
<th>( K_0 ) (mm day(^{-1}))</th>
<th>PWP (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karaj</td>
<td>Mean</td>
<td>3.95</td>
<td>554.72</td>
<td>6.18</td>
<td>0.10</td>
<td>0.06</td>
<td>5.98</td>
<td>0.03</td>
<td>0.0024</td>
<td>0.0173</td>
<td>85.94</td>
</tr>
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<td></td>
<td>Standard deviation</td>
<td>1.73</td>
<td>76.21</td>
<td>0.91</td>
<td>0.06</td>
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<td>450–700</td>
<td>5–8</td>
<td>0.01–0.2</td>
<td>0.01–0.1</td>
<td>2–10</td>
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<td>0.001–0.005</td>
<td>0.04–0.1</td>
<td>70–100</td>
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<td>Lar</td>
<td>Mean</td>
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<td>453.04</td>
<td>5.10</td>
<td>0.19</td>
<td>0.01</td>
<td>9.54</td>
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<td>0.0027</td>
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<td>1.01</td>
<td>2.97</td>
<td>0.08</td>
<td>0.01</td>
<td>0.0007</td>
<td>0.4</td>
<td>0.0002</td>
<td>0.0002</td>
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<td>8.33</td>
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<td>0.13–0.2</td>
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<td>0.08–0.01</td>
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<td>Latyan</td>
<td>Mean</td>
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<td>465.61</td>
<td>5.19</td>
<td>0.18</td>
<td>0.05</td>
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<td>0.0039</td>
<td>0.0551</td>
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<tr>
<td></td>
<td>Standard deviation</td>
<td>1.70</td>
<td>7.79</td>
<td>0.09</td>
<td>0.01</td>
<td>0.025</td>
<td>2.27</td>
<td>0.0207</td>
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<td>5.83</td>
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<td>1–7</td>
<td>450–500</td>
<td>5–5.5</td>
<td>0.14–0.2</td>
<td>0.01–0.02</td>
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<td>0.04–0.1</td>
<td>70–100</td>
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within a greater region. Analysis for the same rainfall-runoff model in different watersheds
the parameters DD parameter meters for the three watersheds: parameter K1 for Karaj watershed,
K2 for Lar watershed, and parameter K2 for Lar watershed. From the large number of the generated ensembles,
the 30 best time series with the highest values of common mean correlation (CMC) were selected for further analysis. An
average of the simulation performance for the selected time series for each watershed is presented in Table 3.
Fig. 5 shows the domain of variation in HBV model parameters that lead to the model maximum simulation performance were obtained for each watershed. From the large number of the generated ensembles, the 30 best time series with the highest values of common mean correlation (CMC) were selected for further analysis. An average of the simulation performance for the selected time series for each watershed is presented in Table 3.

Fig. 7 shows the domain of variation in HBV model parameters that lead to the best simulation performance for Karaj watershed, as an example. The horizontal axes in this figure correspond to the 30 selected best simulated streamflow ensembles. This figure shows that high simulation performance for the watershed rainfall-runoff model can be obtained by a range of values for the parameters rather than by an individual set of values. For some of the parameters, the range of this variation is considerably wide. Fig. 8 compares the simulated streamflow for the watersheds with the observed runoff. The simulated streamflow in this figure is obtained using the best set of the model parameters to run the hydrologic model.

**Uncertain Performance Assessment of Water Supply System**

Ensemble predictions add additional information to the deterministic simulation. Selected time series of runoff generate a bound for possible uncertain streamflow for each watershed. It is important to consider the inflows within these bounds, instead of a single deterministic streamflow, as possible input values to the WSS model. To facilitate the time run of the hydrologic and the system models and to be able to properly map the uncertainty bounds, in addition to the 30 selected best time series, maximum and minimum streamflow time series are built using the generated ensembles (the maximum time series is defined at each time step as the highest streamflow values of the ensembles; the same was done for minimum time series, but with the lowest streamflow values of the ensembles in each time step).

The hydrologic model is initialized (warmed up) based on the observed information for a few days prior to the start of simulation. Therefore, the observed reservoir storage for each reservoir at the beginning of simulation is selected as the initial condition of the reservoir. Uncertain streamflow time series are fed to the SD model of the WSS. By running the system simulation model, water is allocated from different sources based on the governing rules and policies to minimize the shortage in the system. As a result, water-level fluctuations and storage changes in reservoirs can be simulated by the SD model for the modeling time horizon. The ranges of monthly average storage for each reservoir based on the uncertain reservoir inflow time series are presented in Fig. 9. Herein, observed values (shown by solid lines) are estimated based on the simulation of the SD model using observed inflows to the reservoirs, and simulation values show the simulation by the SD model using the sets of the simulated streamflow for watersheds by the HBV model (shown by the dashed line).

This figure depicts how operation of each of these reservoirs is sensitive to the uncertain reservoir inflows associated with hydrologic model parameters’ uncertainty. For example, for the Lar and Latyan reservoirs, the best simulated streamflow scenarios represent similar reservoir storage as the observed ones. However, the wide bound of changes in the reservoir storage in Lar Reservoir shows that if the hydrologic model lacks adequate information about the behavior and characteristics of the catchment, then uncertainty will be largely propagated in WSS simulation. This significantly affects the operation of the system in different months. The propagation of hydrologic model parameters’ uncertainty in simulation of reservoir storage in the Latyan Reservoir is less affected, although the reservoir size is much smaller than Lar and Karaj. For the Karaj and Latyan reservoirs, the simulation of WSS for the observed inflows tends to present water stored in reservoirs closer to the upper bound of the uncertainty range, while for the Lar Reservoir it almost represents the mean reservoir storage. Generally speaking, the propagation of uncertainty, and therefore the size of uncertainty bounds, in reservoir storage is directly associated with the size of the reservoir. Moreover, in smaller reservoirs the observed and deterministic simulations tend toward the upper bound of the uncertainty range and as the size of reservoir increases these values move toward the mean values.
Further, to analyze the performance of the WSS system, the reliability, vulnerability, and resiliency measures are estimated. The output is a range of possible values of these metrics (except resiliency) over 5 years of simulation as shown in Fig. 10.

Based on Fig. 10(a), WSS reliability varies between 0.2 and 1 (at the end of a 5-year period). Any change in reliability could affect making suitable decisions for future planning and operation of Tehran’s WSS. A wide range of reliability values emphasizes the sensitivity of the system’s performance to the uncertainties of streamflow simulations and inflows to the reservoirs. Therefore, it is necessary to take into account the uncertainty analysis of streamflow simulations during the decision-making process, rather than relying on individual deterministic simulations. For Tehran’s WSS, uncertainty in the system’s reliability is considerably high. Ignoring the sources of uncertainty in decision making for the WSS will cause serious errors in operation and planning stages of the system. While the system faces many failure events and has a wide range of reliability, the variations in vulnerability are negligible (between 0.0 and 0.16) [Fig. 10(b)]. An interpretation of the values for these two metrics is that the number of failures in the system can be high, while the degree and magnitude of failures are low. Although the system can fail over many days, the shortage of water for these failures is not significant based on the vulnerability measure. Considering a high and a low range of variations for reliability and vulnerability, respectively, proves that a single-criterion performance assessment cannot be a reliable procedure for decision making of WSSs. The resiliency graph has not been shown here because it is understood from the reliability measure that no failure events have been seen for the observed and high streamflow ensemble time series. Therefore, resiliency cannot be estimated for these
scenarios. However, the results show that the lower bound of the resiliency measure at the end of the 5-year period is 0.016. This shows that if streamflow is underestimated by the hydrologic model or the inflows decrease in the system, Tehran’s WSS will show a nonresilient behavior and the recovery periods in the system would be long. This verifies the importance of multicriteria decision making in management of water resources systems under uncertain hydrologic responses and reservoir inflows.

Summary and Conclusion

An integrated scheme is proposed for the performance assessment of water supply systems. Tehran’s WSS was selected as the realistic case study. The study region includes three watersheds. The rainfall-runoff process in the watersheds was simulated using the HBV hydrologic model. The effects of model parameters’ uncertainty on system performance were then evaluated. A Markov chain

Fig. 7. Range of variation for the HBV model parameters’ values that resulted in generating time series with the highest hydrologic performance
Fig. 8. Comparison of the observed and simulated streamflow for watersheds: (a) Karaj; (b) Lar; (c) Latyan
Fig. 9. Monthly average storage of water in different reservoirs (solid line shows observed and dashed line shows the best simulation): (a) Karaj; (b) Lar; (c) Latyan

Fig. 10. Assessment of Tehran water supply system performance under uncertain streamflow simulation (solid line shows observed and dashed line shows the best simulation): (a) reliability; (b) vulnerability
Monte Carlo–based algorithm was used for this purpose. A system dynamics approach was employed to model interrelationships and complexities in the water supply system. The STELLA software was used to establish the system dynamics model of the system. Three metrics of reliability, vulnerability, and resiliency were used for evaluation of the system performance.

Based on the results, acceptable simulation performance for a hydrologic model can be obtained using a set of probable values for the model parameters rather than individual values. The results of uncertainty analysis of the model parameters indicate that for a watershed some parameters could be well defined, while others may have considerable uncertainty.

The importance of studying the propagation of hydrologic model parameters’ uncertainty is often underrepresented in water resources system analysis. Most studies use a calibrated hydrologic model to produce the inflows to the reservoirs and a WSS to assess the performance of them. However, this research shows that the uncertainty associated with the model parameters and slight changes in their values can often lead to a great change in the performance of a water system, and therefore affect the operation and management of the system. In general, the uncertainty propagation has greater effects on larger reservoirs and systems. The results of this study indicate that system performance could be considerably different when uncertainties in streamflow simulation are taken into account, in comparison with the deterministic analysis of the system. For example, while the number of failure events in the system can be highly increased by changes in the hydrologic model parameters’ values, water shortage in the system is less sensitive to the changes. Also, while the deterministic simulation offers that Tehran’s WSS is resilient, the uncertainty analysis represents long recovery periods for the system, and therefore suggests this system has nonresilient behavior.

The key findings emphasize the importance of incorporating the uncertainty analysis in modeling WSSs and multicriteria performance assessment of the water supply systems. This will help water managers and stakeholders make affirmative decisions based on the informative policies and probable performances of the system. Finally, this study highlights the need for better quality hydrologic modeling, using stochastic and uncertainty simulation of a water system, and multiple performance criteria for assessment of the system performance during uncertain conditions.

References


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