

ESM 121

Water Science
and Management

Spring 2015

Exercise 7:

Interest Rate Formulas and
Incremental Benefit-Cost Method



Samuel Sandoval Solis, PhD

Table of Contents

Objective 1

Interest Rate Formulas 2

 Estimation of Future Value 2

 Estimation of Present Value 5

 Estimation of Annualized Value 7

 Net Benefits Estimation: Benefit - Cost 10

Incremental Benefit – Cost Method 11

Objective

The objective of this exercise is to provide examples to estimate the annualized, future and present cost-benefit values,

This exercise will set the foundation for benefit-cost analysis and engineering economics.

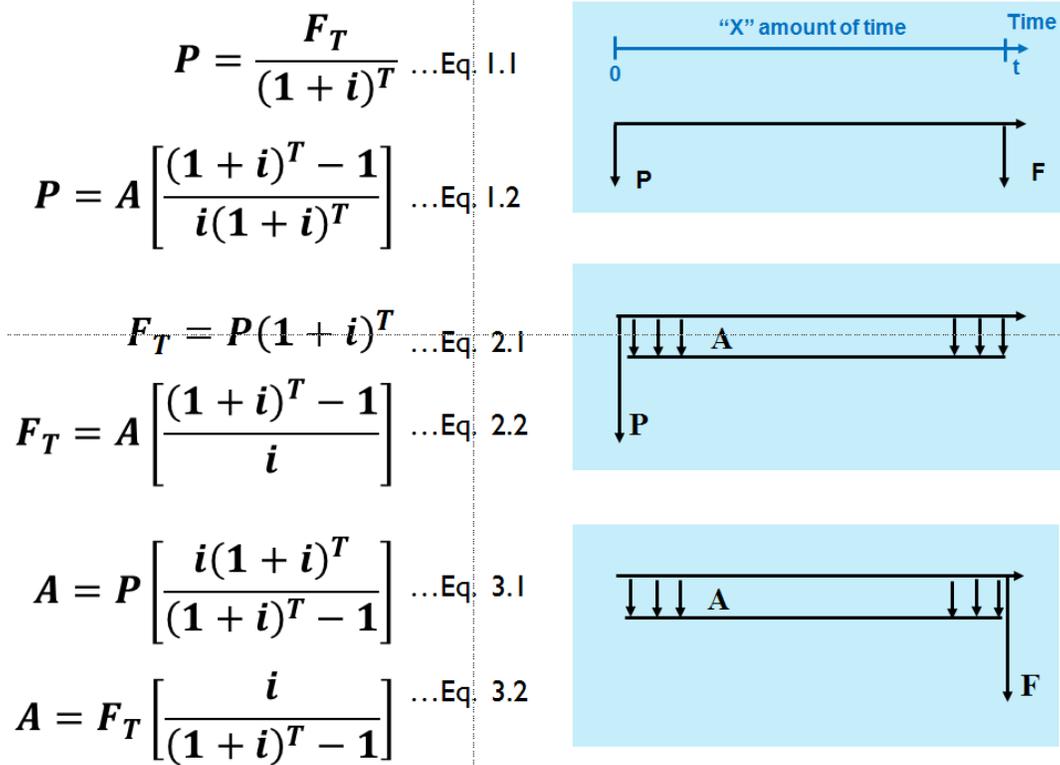


Figure 1 – Interest Rate Formulas

In addition, the objective of this exercise is to provide an example and an exercise of the Incremental Benefit-Cost Method to perform a Benefit-Cost analysis.

Interest Rate Formulas

This first part of the exercise will show you how to use the interest rate formulas shown in Figure 1. In this first section we will start with estimating the future value of benefits or cost.

Estimation of Future Value

Knowing the present value – Worksheet 1.1

Exercise 1.1.a: In 1877, U.S. President Rutherford B. Hayes signed the Desert Land Act, offering land to people for a very low price (\$0.25/acre) with the promise to build the irrigation infrastructure. This act promoted a disproportionate expansion of agriculture land and water consumption in Colorado and New Mexico.

Question: What is a comparable price of \$0.25/acre in current dollars considering an interest rate of 6%?

Response:

This is the data that we know:

P =Land value =	\$0.25	/acre
Interest Rate		
i =	6%	
From :	1877	
To :	2015	
T =	136	years

What we want to know is how much \$0.25 in 1877 is worth in current dollars. So we are going to use equation 2.1.

$$F_t = P(1 + i)^t \dots [2.1]$$

To be turned in: a) What is the cost of \$0.25 in current dollars? b) Can you imagine buying an acre of land in either CO or NM for that price? Is it too much or too little? The current riverfront property price varies from \$5,100/acre to \$1,600/acre. A property in the middle of the back country is about \$730/acre!!! Please elaborate on this.

Exercise 1.1.b: The first Apple Computer called the Apple I went on sale in July 1976 for a retail price of \$666.66. Some say the price was chosen because of the repeating digits but in reality, the first 50 computers were originally sold to the Byte Shop for \$500 who then added a one-third markup to the price.

Question: What is a comparable price of \$666.66 in current dollars considering an interest rate of 5.5%?

Response

The data that we know is:

P =Original Price =	\$666.66
Interest Rate	
i =	5.5%
From :	1976
To :	2015
T =	39 years

What we want to know is how much is worthy \$666.66 1976 dollars in current dollars. So we are going to use equation 2.1.

$$F_t = P(1 + i)^t \dots [2.1]$$

To be turned in: a) What is the cost of \$666.66 in current dollars? b) Current apple computers cost from \$1,500 (13" MacBook Air) to \$2,800 (15" MacBook Retina Display). Was the 1976's price too high or too low?

Knowing the Annualized Cost – Worksheet 1.2

Exercise 1.2. In your voluntary retirement plan, you are considering investing \$500/month for the remaining 30 years that you'll be working, about \$6,000/year. Consider an aggressive fixed interest cost rate of 8% per year.

Question: How much will this constant investment represent in the future?

Response:

Data:

A = Annual savings to retirement=	\$6,000.00	/year
	Interest Rate	
	$i =$	8%
	From :	2015
	To :	2045
T =		30 years

What we want to know is how much a constant \$6,000 savings **per year** is worth after 33 years at an interest rate of 8%. So we are going to use equation 2.2.

$$F_t = A \left[\frac{(1+i)^T - 1}{i} \right] \dots [2.2]$$

To be turned in: a) What is the future retirement saving value (F_{33})?

Estimation of Present Value

Knowing the Future Cost – Worksheet 2.1

Exercise 2.1. In order to improve the water supply, the Pajaro Valley Water Management Agency (PVWMA) is planning to augment the storage capacity at the water treatment facility; they are expecting to increase the "back up" water that can be used during peak periods. The estimated cost in 2020 is about \$6.2 million.

Question: What is the present cost of augmenting the storage capacity if the interest rate considered is 3%?

Response

Data:

Response

F_T = Estimated cost of the project in the Future =	\$6,200,000	
	Interest Rate	
	$i =$	3%
	From :	2015
	To :	2020
	$T =$	5 years

What we want to know is how much 2020's \$6.2 million is worth in present value at an interest rate of 3%. So we are going to use equation 1.1.

$$P = \frac{F_T}{(1 + i)^T} \dots [1.1]$$

To be turned in: What is the current value of augmenting the water storage at the recycling facility (P)?

Knowing the Annualized Cost – Worksheet 2.2

Exercise 2.2. When a system is improved, some of the benefits come in the reduction of "O&M" (operation and maintenance). Let's say replacing 8 pumps in a groundwater extraction facility represents a reduction in the O&M cost of \$500/pump every year.

Question: What is the present value of the savings provided by the pumps if their life span is 20 years at an interest rate of 4.5%?

Response

Data:

# of pumps =	8	pumps
Savings per pump =	\$500	
A = Savings per year =	\$4,000	/year
Interest Rate		
	$i =$	4.5%
T =	20	years

What we want to know is how much \$4,000 of savings per year are worth during 20 years in present value (2015's dollars) at an interest rate of 8%. So we are going to use equation 1.2.

$$P = A \left[\frac{(1 + i)^T - 1}{i(1 + i)^T} \right] \dots [1.2]$$

To be turned in: a) What is the present value of having benefits of \$4,000/year during the next 20 years? b) What are the benefits that each pump is providing (P/8)? c) If you would like to make a decision about whether or not to replace a pump, how much will a pump have to cost in order to be economically feasible to replace? Hint: if the benefits per pump (incise "b") are less than or equal to the cost of a pump, then it is economically feasible to buy a new pump.

Estimation of Annualized Value

Knowing the Present Cost – Worksheet 3.1

Exercise 3.1. You want to buy a Mini Cooper Convertible (red preferably!) and you are wondering how much it will cost you. The MSRP (Manufacturer's Suggested Retail Price) on this model is \$27,350, the APR (Annual Percentage Rate) that the dealer is offering you is 0.9% for up to 60 months*.

Question: What are the annual payment and monthly payment that you have to pay to cover the capital cost (for the \$27,350) of the car?

Response

Data:

MSRP (Manufacturer's Suggested Retail Price)		
MSRP =	\$27,350	
APR (Annual Percentage Rate)		
APR =	0.9%	
For up to :	60	months *
T =	5	years *

What we want to know first is the annual amount that you have to pay if the Present Value (P) of the car is \$27,350 at an annual interest rate of 0.9% over 5 years (60 months). So we are going to use equation 3.1.

$$A = P \left[\frac{i(1+i)^T}{(1+i)^T - 1} \right] \dots [3.1]$$

To be turned in: a) What is the annual payment (A) for the capital cost of a red convertible mini cooper? b) What is the monthly payment? Hint: Divide incise “a” by 12

If we look at the small letters of the advertisement, they said: “Compound interest of **\$17.50/month for every \$1,000 financed!!!!**” Is this too high or too low? So, if you are not doing any down payment, the following calculation tells you the monthly payment that you are going to pay for the whole capital cost of the vehicle.

Data:

Amount Financed:	\$27,350	
number of \$1,000 financed:	27.35	
Compound Interest:	\$17.50	/month per \$1,000 financed
Charge for Compound Interest	\$479	/month

You are going to be paying ~\$479 per month for the “Compound interest”!!!!

To be turned in: a) What is the *total monthly payment* that you will have to pay (Capital cost payment + Compound interest payment)? b) What is the total amount that you will have to pay at the end of the fifth year? Hint: Incise “a” times 60. c) Is this amount twice as much as the present value of the car?

Knowing the Future Value – Worksheet 3.2

Exercise 3.2. The Pajaro Valley Water Management Agency (PVWMA) is expected to upgrade the Harkins Slough Recharge Project. Basically, this is an active water bank. In wet periods, water is pumped out of Harkins slough and conveyed to a “replenishment facility” or in more common words, a pond. Water in the pond infiltrates to the aquifer where the water is stored in the groundwater bank. Then, in drought years, water is pumped out of the bank, which is the aquifer underneath the replenishment facility (the pond), and moved into the water supply delivery system. The capital cost to upgrade this recharge facility by buying more pumps is estimated to be \$1,000,000 in 2020.

Question: What is the annual cost that this facility represents in current dollars at an interest rate of 3%?

Response

Data:

	Capital Cost	
	$F =$	\$1,000,000
		Interest Rate
	$i =$	3.0%
Year of Implementation =		2020
	$T =$	5 years *

What we want to know is the annual cost in today's dollar of \$1 million in 2020 at an interest rate of 3%. So we are going to use equation 3.2.

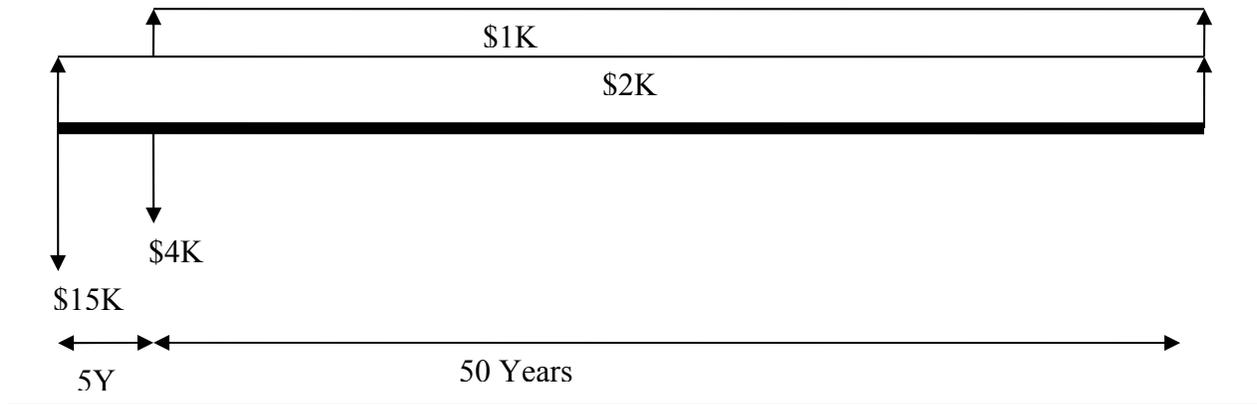
$$A = F_T \left[\frac{i}{(1+i)^T - 1} \right] \dots [3.2]$$

To be turned in: a) What is the annual cost (A) for improving this facility? b) If the increased yield for this facility is expected to be 1,000 AF/year, what is the unit cost per acre-foot that should be charged? Hint: divide incise “a” by 1,000 AF/year

Net Benefits Estimation: Benefit - Cost

Now, you will have to work on the following exercise on your own:

You are working with the manager of an irrigation facility who is interested in installing a more efficient pumping system. The proposed system costs \$15,000 and you estimate that it will reduce the annual utility costs by \$2,000. After five years, you expect to upgrade the system for \$4,000. This upgrade is expected to further reduce utility costs by \$1,000 annually. The annual effective interest rate is 7% and the life of the system after upgrade is 50 years.



To be turned in: a) What is the Present Value of the costs (efficient pumping system and upgrade cost)? b) What is the Present Value of the benefits? c) What is the Present Value of the net benefits (benefits – costs) of the investment in the system?

Incremental Benefit – Cost Method

Exercise 1.

A flood control district would like to decide which of the projects (or combination of projects) is more cost-benefit effective. The alternatives include dam A, dam B and a levee system C. Using the incremental method identify which is the preferred alternative.

Project	Life (years)	Capital Cost (million \$)	Annual Operation and Maintenance (million \$)	Average Annual Flood Damages (million \$)
Do nothing		0	0	2.0
A (dam)	80	6	0.09	1.1
B (dam)	80	5	0.08	1.3
C (levee)	60	6	0.10	0.7
AB		11	0.17	0.9
AC		12	0.19	0.4
BC		11	0.18	0.5
ABC		17	0.27	0.25

First, let's estimate the cost of each project. In the following table, Column 1 is calculated using Equation 3.1; P is the capital cost, i is the interest rate ($i=4\%$) and T is the expected time in years to pay for the infrastructure. First, individual annual capital cost of alternatives A, B and C are estimated. Then, for alternative AB, the annual capital costs of A and B are summed to find the annual capital cost of AB ($0.265 = 0.251 + 0.209$). This same calculation applies for the annual capital cost of the remaining combined alternatives.

$$A = P \left[\frac{i(1+i)^T}{(1+i)^T - 1} \right] \dots [3.1]$$

The annual operation and maintenance (O&M) is given data and the total annual cost is the sum of the annual capital cost plus the annual O&M (Column 3 equals Column 1 + Column 2). Notice that the alternatives have been ranked to determine which are the least expensive (Do nothing, followed by Dam B) to the most expensive (Alternative ABC, building all the infrastructure).

Project	Life (years)	(1) Using Eq. 3.1 Annual Capital Cost (million \$)	(2) Annual Operation and Maintenance (million \$)	(3) (3 = 1 + 2) Total Annual Costs (million \$)	(4) Rank
Do nothing		0	0	0	1
A (dam)	80	0.251	0.090	0.341	3
B (dam)	80	0.209	0.080	0.289	2
C (levee)	60	0.265	0.100	0.365	4
AB		0.460	0.170	0.630	5
AC		0.516	0.190	0.706	7
BC		0.474	0.180	0.654	6
ABC		0.725	0.270	0.995	8

Note. The rank in column (4) is based on ordering the total annual cost from the least expensive [Do-nothing alternative, rank 1] to the most expensive [Alternative ABC, rank 8]

Now, let's estimate the Benefits by subtracting the annual flood damage of each alternative from the flood damage caused by the "Do nothing" alternative (Column 3 = Column 1 – Column 2),

Project	(1) Do-nothing Avg Annual Flood Damage (million \$)	(2) Average Annual Flood Damages (million \$)	(3) (3 = 1 – 2) Total Annual Benefits (million \$)
Do nothing	2.0	2.0	0
A (dam)	2.0	1.1	0.9
B (dam)	2.0	1.3	0.7
C (levee)	2.0	0.7	1.3
AB	2.0	0.9	1.1
AC	2.0	0.4	1.6
BC	2.0	0.5	1.5
ABC	2.0	0.25	1.75

Then, let's build a summary table where the costs and benefits are summarized and the alternatives are organized from least expensive to most expensive. The Benefit-Cost ratio should be calculated (Benefit/Cost), and if the benefit cost ratio is less than 1, that alternative should be discarded for the following analysis (if $B/C < 1$ then discard the alternative).

Project	Total Annual Benefits (million \$)	Total Annual Costs (million \$)	Benefit/Cost	Rank
Do nothing	0.00	0.00		1
B (dam)	0.70	0.29	2.42	2
A (dam)	0.90	0.34	2.64	3
C (levee)	1.30	0.37	3.56	4
AB	1.10	0.63	1.75	5
BC	1.50	0.65	2.29	6
AC	1.60	0.71	2.27	7
ABC	1.75	1.00	1.76	8

Note The rank in the last column is based on ordering the total annual cost from the least expensive [Do-nothing alternative, rank 1] to the most expensive [Alternative ABC, rank 8]

Now, let's start applying the incremental Benefit-Cost method. The first analysis always starts with the Do-nothing alternative (θ) compared to the least expensive alternative (B(dam)); in this case the *Current Best Alternative* is the Do-nothing alternative and the *Contender* alternative is alternative B (Dam b). Benefits (B) and Costs (C) are placed in columns 1 and 2, respectively. Then, the benefit cost ratio is estimated in column 3 (B/C). The incremental benefits (ΔB) are calculated, and only for this first case is this value equal to the least expensive alternative benefits ($\Delta B=B$). The incremental costs (ΔC) are calculated, and again only for this first case is this value equal to the least expensive alternative costs ($\Delta C=C$). The incremental Benefit-Cost ratio is calculated ($\Delta B/ \Delta C$), if this ratio is higher than 1, then the *Contender* is the new *Current Best* alternative, which is what happens in this case, Alternative B is the new *Current Best* alternative, this is why in column 7 is written " $B > \theta$ ", meaning Alternative B is preferred than Do-nothing alternative.

Compare	Project	(1) B (mil\$)	(2) C (mil\$)	(3) Benefit/Cost (B/C)	(4) ΔB (mil\$)	(5) ΔC (mil\$)	(6) $\Delta B/\Delta C$ (mil\$)	(7) Decision
	Do Nothing	---	---					
$\theta - B$	B (dam)	0.70	0.29	2.42	0.7	0.29	2.42	$B > \theta$

Now, let's compare alternative B, which is the *current best*, with alternative A, *the contender*, which is the following more expensive alternative after B. First, we place the Benefits (B) and Costs (C) in columns 1 and 2, respectively. Then the benefit-cost ratio is estimated in column 3 (B/C). The incremental benefits (ΔB) are calculated, in this case the incremental cost is the difference between the contender benefits of alternative A minus the current best benefits from alternative B ($0.2 = 0.9 - 0.7$). Similarly, the incremental costs (ΔC) are calculated by the difference between the contender costs of alternative A minus the current best costs from alternative B ($0.05 = 0.34 - 0.29$). The incremental Benefit-Cost ratio is calculated ($\Delta B/ \Delta C$), if

this ratio is higher than 1, then the Contender is the new Current Best alternative, which is what happens in this case, Alternative A is the new Current Best alternative, this is why in column 7 we have written “A > B,” meaning Alternative A is preferred over Alternative B.

Compare	Project	(1) B (mln\$)	(2) C (mln\$)	(3) Benefit/Cost (B/C)	(4) ΔB (mln\$)	(5) ΔC (mln\$)	(6) $\Delta B/\Delta C$ (mln\$)	(7) Decision
	Do Nothing	---	---					
$\theta - B$	B (dam)	0.70	0.29	2.42	0.7	0.29	2.42	B > θ
B - A	A (dam)	0.90	0.34	2.64	0.2	0.05	3.86	A > B

Then Alternative C will be compared against Alternative A. In this case, Alternative C turns out to be the preferred alternative, the new “Current Best” over Alternative A. Then alternative C is evaluated against alternatives AB, BC, AC and ABC, and in all these cases alternative C will prevail as the preferred alternative over the remaining alternatives, the incremental benefit cost ration in all these comparisons is less than 1 (Column 6). **Thus, Alternative C is the alternative that provides the most efficient Benefit-Cost ratio.**

Compare	Project	(1) B (mln\$)	(2) C (mln\$)	(3) Benefit/Cost (B/C)	(4) ΔB (mln\$)	(5) ΔC (mln\$)	(6) $\Delta B/\Delta C$ (mln\$)	(7) Decision
	Do Nothing	---	---					
$\theta - B$	B (dam)	0.70	0.29	2.42	0.7	0.29	2.42	B > θ
B - A	A (dam)	0.90	0.34	2.64	0.2	0.05	3.86	A > B
A - C	C (levee)	1.30	0.37	3.56	0.4	0.02	16.44	C > A
C - AB	AB	1.1	0.63	1.75	-0.2	0.26	-0.76	C > AB
C - BC	BC	1.5	0.65	2.29	0.2	0.29	0.69	C > BC
C - AC	AC	1.6	0.71	2.27	0.30	0.34	0.88	C > AC
C - ABC	ABC	1.75	0.99	1.76	0.45	0.63	0.71	C > ABC

Exercise 2. A flood control district can construct a number of alternative control works to alleviate the flood pattern in the district. These alternatives include dam A, dam B, and a levee system C. The levee system can be built alone or in combination with dam A or B. Both dams cannot be built together but either one can function alone. The lifespan of each dam is 80 years and the lifespan of the levee system is 60 years. The cost of capital is 6 percent. Information on total investment, operation and maintenance costs, and average annual flood damage is given below. What form of flood control would be the most economical?

Project	Life (years)	Total Investment (million \$)	Annual Operation and Maintenance (million \$)	Average Annual Flood Damages (million \$)
A (dam)	80	6.2	0.093	1.10
B (dam)	80	5.3	0.089	1.40
C (levee)	60	6.7	0.110	0.80
AC		12.9	0.203	0.4
BC		12.0	0.199	0.5
Do nothing		0	0	2.15

- a) Estimate the Annual Cost (A) for the total investment of each alternative (Column 3) using the interest rate formulas. Then estimate the Total Annual Cost (Column 5) by adding the Annual Investment (Column 3) plus the Annual Operation and Maintenance (Column 4).

(1) Project	(2) Life (years)	(3) Annual Investment (million \$ / year)	(4) Annual Operation and Maintenance (million \$ / year)	(5) Total Annual Costs (million \$)
A (dam)	80		0.093	
B (dam)	80		0.089	
C (levee)	60		0.110	
AC			0.203	
BC			0.199	
Do nothing		0	0	0

- b) Estimate the benefits (column 4) by subtracting the Do-nothing damages (\$2.15 million/year – Column 2) to each of the average annual flood damages (Column 3)

(1) Project	(2) Do Nothing Flood Damages (million \$/year)	(3) Annual Investment (million \$ / year)	(4) Average annual benefits (million \$ / year)
A (dam)	2.15	1.10	
B (dam)	2.15	1.40	
C (levee)	2.15	0.8	
AC	2.15	0.4	
BC	2.15	0.5	
Do nothing	2.15	2.15	

- c) Estimate the most economical flood control alternative using the Incremental Benefit-Cost Method.

Compare	Project	Benefit (B) (mln\$)	Cost (C) (mln\$)	Benefit/Cost (B/C)	ΔB (mln\$)	ΔC (mln\$)	$\Delta B/\Delta C$ (mln\$)	Decision

Remember:

1. For each alternative
 - Define consequences
 - Estimate value of consequences, i.e. Costs and Benefits
 - Calculate B/C ratios, Discard any with $B/C < 1$
2. Order alternatives: **Lowest to highest cost**
3. Select lowest cost alternative as “Current Best”
 - Next higher cost alternative is “Contender”
4. Compare “Best” to “Contender”
 - Compute DB/DC , If $DB/DC > 1$, Contender becomes Best
5. Repeat Step 4 for all alternatives
6. Final “Current Best” is “Preferred Alternative”